CSC 240

LOGIC 2

PROPOSITIONAL LOGIC

Proposition: A statement that is either true or false.

Tony Stark is rich.

Captain America is the son of Odin.

Connectives: Things that connect or modify propositions.

$\neg$ (not) $\wedge$ (and) $\vee$ (or) $\rightarrow$ (implies)

$\longleftrightarrow$ (biconditional, "if and only if")

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Predicates: Produce a proposition based on the properties of an object.

PREDICATES

predicates produce propositions

Avenger(TonyStark) = True

Avenger(Superman) = False

Relatives(Mario, Luigi) = True

Relatives(Katniss, Gale) = False

3 = 4 = False

5 < 14 = True

predicates produce propositions

Avenger(TonyStark) → FailToWin(Loki)

Even(x) ⟷ EvenlyDivisibleByTwo(x)

FIRST ORDER LOGIC

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Avenger(TonyStark) Relatives(Mario, Luigi)

$3 < 12$ $3 + 2 = 5$

Functions: Produce on object based on the properties of another object.

functions produce objects

SecretIdentity(SpiderMan) = Peter Parker (MCU)

SecretIdentity(Superman) = Clark Kent

ColorOf(BrotherOf(Mario)) = Green

ColorOf(BrotherOf(Mario)) = ColorOf(WeaknessOf(Superman))

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FIRST ORDER LOGIC

	Input	Output	Example
Connectives	Proposition(s)	A Proposition	$x \in A \rightarrow x \in A \cap B$
Predicates	Object(s)	A Proposition	IsFood(Carrot)
Functions	Object(s)	An Object	Breakfast(Yesterday)

What is this?

$x < 12$

Predicates must generate a proposition,
but we can’t unless we know what x is.

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Quantifiers: Describe the "domain" of a predicate or function.

QUANTIFIERS

∃ The Existential Quantifier

"There Exists"

∃x. (expression)

"There Exists an x for which (expression) is true."

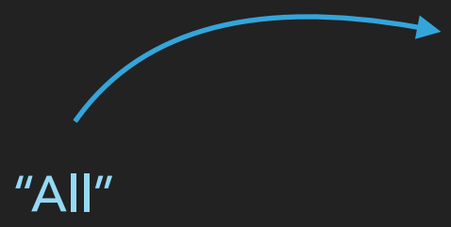
"Results in a proposition"

∃x. (x < 12) = true

∃x. (x < 12 ∧ x > 12) = false

∃x. (IsStudent(x) ∧ HasJob(x)) = true

QUANTIFIERS



∀ The Universal Quantifier

∀x. (expression)

∀x. (IsPerson(x) → (EatsTooMuch(x) ∧ WillBeSick(x))) = true

∀s. (IsStudent(s) → (DoesHomework(s) ∨ WillFail(s))) = true

QUANTIFIERS

Not all quantifier expressions result in true propositions.

$$\forall p.\ (\ \text{IsAvenger}(p) \rightarrow \text{HasSuperPower}(p)\ )$$

true or false?

To prove a universal quantifier false, you only need one counterexample.

$$\exists p.\ (\ \text{IsAvenger}(p) \wedge \text{HasSuperPower}(p)\ )$$

true or false?

To prove a existential quantifier true, you only need one positive example.

WHO CARES?

First order logic makes it easy to manipulate statements.

To negate a statement, translate it into first order logic, negate that, then translate it back.

If you need to prove something by contrapositive, translate it into first order logic, take the contrapositive, then translate it back.

Warning: Translating into first order logic can be tricky, we'll go through some examples next time.

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$\exists$x. (expression) $\forall$x. (expression)