CSC 240

PROOFS 4

P => Q

- Assume the negation of the statement (P => ~Q)
- Start with either P or ~Q
- Attempt to move towards the other statement until you arrive at a contradiction.

If you play with fire, you'll get burned.

I didn't get burned, so I didn't play with fire.

If it rains, you'll get wet.

You didn't get wet, so it didn't rain.

P => Q

NOT Q => NOT P

INDIRECT PROOFS - PROOF BY CONTRAPOSITIVE

If you eat too much, you'll feel sick.

If you don't feel sick, you didn't eat too much.

	P	Q	P => Q	~P	~Q	~Q => ~P	
You ate too much. You feel sick.	T	T	T	F	F	T	You feel sick. You ate too much.
You ate too much. You don't feel sick.	T	F	F	F	T	F	You don't feel sick. You ate too much.
You didn't eat too much. You feel sick.	F	T	T	T	F	T	You feel sick. You didn't eat too much.
You didn't eat too much. You don't feel sick.	F	F	T	T	T	T	You don't feel sick. You didn't eat too much.

If you eat too much, you'll feel sick.

logically equivalent

If you don't feel sick, you didn't eat too much.

INDIRECT PROOFS - PROOF BY CONTRAPOSITIVE

P => Q

If $A \subseteq B$, then $(A \cap B) = A$

P

Q

~Q => ~P

If $(A \cap B) \neq A$, then $A \nsubseteq B$

~P

~Q

A: $(A \cap B) \neq A$

A1: $x \in A$ and $x \notin (A \cap B)$

What it means to be "not equal"

A2: $x \notin B$

What "intersection" means

B1: $x \in A$ and $x \notin B$

What it means to not be a "subset" of a set.

What does it mean for a set NOT to be a subset of another set?

B: $A \nsubseteq B$

INDIRECT PROOFS - WRITING THE PROOF WITH WORDS

If $A \subseteq B$, then $(A \cap B) = A$

If $(A \cap B) \neq A$, then $A \not\subseteq B$

A: $(A \cap B) \neq A$

A1: $x \in A$ and $x \notin (A \cap B)$

B1: $x \in A$ and $x \notin B$

B: $A \not\subseteq B$

We will prove the *contrapositive*, i.e. for sets A and B

$$(A \cap B) \neq A \;\Rightarrow\; A \not\subseteq B \qquad (1)$$

From (1) we see that there exists an element *x* such that

$$x \in A \qquad (2)$$

and

$$x \notin (A \cap B)\,. \qquad (3)$$

From (3) we see that either

$$x \notin A \qquad (4)$$

or

$$x \notin B\,. \qquad (5)$$

However, (4) contradicts (2), therefore

$$x \in A \;\text{ and }\; x \notin B. \qquad (6)$$

Hence

$$A \not\subseteq B. \;\blacksquare$$

INDIRECT PROOFS - WRITING THE PROOF WITH LATEX

```latex
We will prove the contrapositive, i.e. for sets A and B

\begin{equation}
\label{contra}
  (A \cap B) \ne A  \Rightarrow  A \nsubseteq B
\end{equation}

From \ref{contra} we see that there exists an element $x$ such that

\begin{equation}
\label{x-A}
  x \in A
\end{equation}

and

\begin{equation}
\label{x-intersect}
  x \notin (A \cap B).
\end{equation}

From \ref{x-intersect} we that either

\begin{equation}
\label{x-intersect-A}
  x \notin A
\end{equation}

or

\begin{equation}
\label{x-intersect-B}
  x \notin B.
\end{equation}

However, \ref{x-intersect-A} contradicts \ref{x-A},
therefore

\begin{equation}
\label{x-intersect-final}
  x \in A \land x \notin B.
\end{equation}

Hence

\begin{equation}
\label{final}
  A \nsubseteq B. \blacksquare
\end{equation}
```

PROOF TECHNIQUE COMPARISON

Direct Proof / Forward Backwards Method

Given P => Q

Assume P and Q are true.

Work Forwards from P and Backwards from Q until you meet in the middle.

Proof by Contradiction

Given P => Q

Assume P and ~Q are true.

Work Forwards from P and ~Q until a contradiction is found.

Proof by Contrapositive

Given P => Q

Assume ~Q and ~P are true.

Work Forwards from ~Q and Backwards from ~P until you meet in the middle.