CSC 240

PROOFS 3

P => Q

- Assume the negation of the statement (P => ~Q)
- Start with either P or ~Q
- Attempt to move towards the other statement until you arrive at a contradiction.

INDIRECT PROOFS - PROOF BY CONTRADICTION

P => Q   For any integer, if n^2 is odd, then n is odd.

P

Q

P => ~Q   For any integer, if n^2 is odd, then n is even.

A: n is even

A1: $n = 2k$ What it means to be "even"

A2: $n^2 = (2k)^2$ Some algebra

$= 4k^2$

$= 2(2k^2)$

A3: n^2 is even   What it means to be "even"

Contradiction!

INDIRECT PROOFS - WRITING THE PROOF WITH WORDS

For any integer, if n^2 is odd, then n is odd.

For any integer, if n^2 is odd, then n is even.

A: n is even

A1: $n = 2k$

A2: $n^2 = (2k)^2$
$= 4k^2$
$= 2(2k^2)$

A3: n^2 is even

Suppose for the sake of contradiction, that *n* is even.

Since *n is even*, there exists some integer *k* such that

$$n = 2k. \qquad (1)$$

Squaring (1) we see that

$$\begin{aligned} n^2 &= (2k)^2 \qquad (2) \\ &= 4k^2 \\ &= 2(2k^2). \end{aligned}$$

From (2) we see that there is an integer *a*, where $a = 2k^2$ such that

$$n^2 = 2a. \qquad (3)$$

However, (3) implies that n^2 is even, which is a contradiction. ■

DIRECT PROOFS - WRITING THE PROOF WITH LATEX

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Suppose for the sake of contradiction, that $n$ is even.

Since $n$ is even, there exists some integer $k$, such that

\begin{equation}
\label{contra-k}
  n = 2k.
\end{equation}

Squaring \ref{contra-k}, we see that

\begin{equation}
\label{contra-algebra}
  \begin{split}
    n^{2} & = (2k)^{2}\\
    & = 4k^{2}\\
    & = 2(2k^{2}).
  \end{split}
\end{equation}

From \ref{contra-algebra} we see that there is an integer $a$, where
$a = 2k^{2}$ such that

\begin{equation}
\label{contra-final}
  n^{2} = 2a.
\end{equation}

However, \ref{contra-final} implies that $n^{2}$ is even, which is a contradiction.
${\blacksquare}$
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INDIRECT PROOFS - PROOF BY CONTRADICTION

P => Q $A \cap (B - A) = \varnothing$

P

Q

P => ~Q $A \cap (B - A) \neq \varnothing$

A: Let $x \in A \cap (B - A)$ — Since $A \cap (B - A)$ isn't empty

A1: $x \in A$ and $x \in (B - A)$ — What "intersection" means

A1: $x \in B$ and $x \notin A$ — What "difference" means

Contradiction!

INDIRECT PROOFS - WRITING THE PROOF WITH WORDS

$A \cap (B - A) = \varnothing$

$A \cap (B - A) \neq \varnothing$

A: Let $x \in A \cap (B - A)$

A1: $x \in A$ and $x \in (B - A)$

A1: $x \in B$ *and* $x \notin A$

Let us assume the original statement is false.

Therefore

$$A \cap (B - A) \neq \varnothing. \tag{1}$$

Then there exists an element *x* such that

$$x \in A \cap (B - A) \tag{2}$$

From (2) we see that

$$x \in A \tag{3}$$

and

$$x \in (B - A) \tag{4}$$

From (4) we see that

$$x \in B \tag{5}$$

and

$$x \notin A \tag{6}$$

Which is a contradiction of (3), therefore it must be that

$$A \cap (B - A) = \varnothing. \blacksquare$$

DIRECT PROOFS - WRITING THE PROOF WITH LATEX

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Let us assume the original statement is false.Therefore

\begin{equation}
\label{inverse}
    A \cap (B - A) \neq \emptyset.
\end{equation}

Then there exists an element x such that

\begin{equation}
\label{x-is}
    x \in A \cap (B - A)
\end{equation}

From \ref{x-is} we see that
\begin{equation}
\label{member-a}
    x \in A
\end{equation}

and

\begin{equation}
\label{member-diff}
    x \in (B - A)
\end{equation}

From \ref{member-diff} we see that
\begin{equation}
\label{member-b}
    x \in B
\end{equation}

and

\begin{equation}
\label{not-member-a}
    x \notin A
\end{equation}

However, \ref{not-member-a} is a contradiction of \ref{member-a},
therefore it must be that
\begin{equation}
\label{final}
    A \cap (B - A) = \emptyset.\blacksquare
\end{equation}
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