CSC 240

PROOFS 2

- Ask a key question.
- Answer the key question abstractly.
- Transform the answer into specifics that match the problem.

DIRECT PROOFS - FORWARD BACKWARD METHOD

For **any** integers m and n, if m and n are odd, then m + n is even.

To show this is true for *any* integers, we have to choose *arbitrary* integers.

arbitrary | ˈärbəˌtrerē |

adjective

based on random choice or personal whim, rather than any reason or system: *his mealtimes were entirely arbitrary*.

- (of power or a ruling body) unrestrained and autocratic in the use of authority: *arbitrary rule by King and bishops has been made impossible*.
- *Mathematics* (of a constant or other quantity) of unspecified value.

DIRECT PROOFS - FORWARD BACKWARD METHOD

For any integers m and n, if m and n are odd, then m + n is even.

A

B

A: m and n are arbitrary odd integers

A1: $m = 2a + 1$
$n = 2b + 1$

What it means to be "odd"

A2: $m + n = (2a + 1) + (2b + 1)$
$= (2a + 2b + 2)$
$= 2(a + b + 1)$

Some algebra…

B1: $m + n = 2k$

What it means to be "even"

B: m + n is even

What does it mean for a number to be even?

DIRECT PROOFS - WRITING THE PROOF WITH WORDS

A: m and n are arbitrary odd integers

A1: $m = 2a + 1$

$n = 2b + 1$

A2: $m + n = (2a + 1) + (2b + 1)$

$= (2a + 2b + 2)$

$= 2(a + b + 1)$

B1: $m + n = 2k$

B: m + n is even

Let m and n be arbitrary odd integers.

Since *m* and *n* are odd, there exist some integers *a* and *b* such that

$$m = 2a+1 \qquad (1)$$

and

$$n = 2b + 1 \qquad (2)$$

Adding (1) and (2), we see that

$$\begin{aligned} m+n &= (2a+1) + (2a+2) \qquad (3) \\ &= (2a + 2b + 2) \\ &= 2(a + b + 1) \end{aligned}$$

From (3) we see that there is an integer k, where k = (a + b + 1) such that

$$m + n = 2k. \qquad (4)$$

Therefore, m + n is even. ■

DIRECT PROOFS - WRITING THE PROOF WITH LATEX

```latex
Let $m$ and $n$ be arbitrary odd integers. Since $m$ and $n$ are odd, there exist some integers
$a$ and $b$ such that

\begin{equation}
\label{m-odd}
  m = 2a + 1
\end{equation}

and

\begin{equation}
\label{n-odd}
  n = 2b + 1.
\end{equation}

Adding \ref{m-odd} and \ref{n-odd}, we see that

\begin{equation}
\label{mn-combined}
  \begin{split}
    m+n & = (2a + 1) + (2a + 2)\\
    & = (2a + 2b + 2)\\
    & = 2(a + b + 1).
  \end{split}
\end{equation}

From \ref{mn-combined} we see that there is an integer $k$ where $k = (a + b + 1)$ such that

\begin{equation}
\label{mnk}
  m + n = 2k.
\end{equation}

Therefore, $m + n$ is even. $\blacksquare$
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The split environment allows you to align multiline equations.

The \\ indicates the end of each line.

The & indicates where the lines should line up.