CSC 240

PROOFS 1

- Ask a key question.
- Answer the key question abstractly.
- Transform the answer into specifics that match the problem.

DIRECT PROOFS - FORWARD BACKWARD METHOD

If n is an even integer, then n^2 is even.

A

B

A: n is an even integer

A1: $n = 2k$ — What it means to be "even"

A2: $n^2 = (2k)^2$ — Some algebra...

$= (2k)(2k)$

$= 4k^2$

$= 2(2k^2)$

B1: $n^2 = 2m$ — What it means to be "even"

B: n^2 is even

What does it mean for a number to be even?

DIRECT PROOFS - WRITING THE PROOF WITH WORDS

A: n is an even integer

A1: $n = 2k$

A2: $n^2 = (2k)^2$

$= (2k)(2k)$

$= 4k^2$

$= 2(2k^2)$

B1: $n^2 = 2m$

B: n^2 is even

Let n be an even integer.

Since n is even, there's some integer k such that

$$n = 2k. \qquad (1)$$

From (1) we see that

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2). \qquad (2)$$

From (2) we see that there is an integer m, where $m = 2k^2$ such that

$$n^2 = 2m. \qquad (3)$$

Therefore, n^2 is even. ■

The end of proof marker.
Q.E.D.
"quod erat demonstrandum"
(what was to be demonstrated)

DIRECT PROOFS - WRITING THE PROOF WITH LATEX

```latex
\subsection{Theorem}
  If $n$ is an even integer, then $n^{2}$ is even.

\subsection{Proof}
  Let $n$ be an even integer. Since $n$ is even, there's  some integer $k$ such that

  \begin{equation}
  \label{even1}
    n = 2k.
  \end{equation}

  From \ref{even1}, we see that

  \begin{equation}
  \label{even2}
    n^{2} = (2k)^{2} = 4k^{2} = 2(2k^{2}).
  \end{equation}

  From \ref{even2}, we see that there is an integer $m$ where $m = 2k$ such that

  \begin{equation}
  \label{even3}
    n^{2} = 2m.
  \end{equation}

  Therefore, $n^{2}$ is even. $\blacksquare$
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DIRECT PROOFS - FORWARD BACKWARD METHOD

If A, B, and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

A: Let $x \in A \cup (B \cap C)$

A1: Then either $x \in A$ or $x \in (B \cap C)$

A2: Then either $x \in A$ or ($x \in B$ and $x \in C$)

Either…or = Proof by Cases

B

What "union" means

What "intersection" means

Choose either statement to be our starting point and work towards the other one

B2: For every element $x \in A \cup (B \cap C)$, $x \in (A \cup B) \cap (A \cup C)$

What it means for sets to be subsets of each other.

B1: $(A \cup (B \cap C)) \subseteq ((A \cup B) \cap (A \cup C))$ and $((A \cup B) \cap (A \cup C)) \subseteq (A \cup (B \cap C))$

What it means for sets to be equal.

B: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

What does it mean a set to be a subset of another set?

What does it mean for two sets to be equal?

A2: Then either $x \in A$ or ($x \in B$ and $x \in C$)

Case 1: $x \in A$

1.1: If $x \in A$, then $x \in (A \cup B)$ — What "union" means

1.2: If $x \in A$, then $x \in (A \cup C)$ — What "union" means

1.3: If $x \in (A \cup B)$ and $x \in (A \cup C)$, then $x \in (A \cup B) \cap (A \cup C)$ — What "intersection" means

Case 2: $x \in B$ and $x \in C$

2.1: If $x \in B$ then $x \in (A \cup B)$ — What "union" means

2.2: If $x \in C$ then $x \in (A \cup C)$ — What "union" means

2.3: If $x \in (A \cup B)$ and $x \in (A \cup C)$, then $x \in (A \cup B) \cap (A \cup C)$ — What "intersection" means

B2a: In every case, $x \in (A \cup B) \cap (A \cup C)$

B2: For every element $x \in A \cup (B \cap C)$, $x \in (A \cup B) \cap (A \cup C)$

If A, B, and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Does $A \subseteq B$ necessarily mean $B \subseteq A$?

A: Let $x \in A \cup (B \cap C)$

A1: Then either $x \in A$ or $x \in (B \cap C)$

A2: Then either $x \in A$ or ($x \in B$ and $x \in C$)

A3: Proof by Cases 1 and 2

B2: For every element $x \in A \cup (B \cap C)$, $x \in (A \cup B) \cap (A \cup C)$

C: Let $x \in (A \cup B) \cap (A \cup C)$

?

B2: For every element $x \in A \cup (B \cap C)$, $x \in (A \cup B) \cap (A \cup C)$

B1: $(A \cup (B \cap C)) \subseteq ((A \cup B) \cap (A \cup C))$ and $((A \cup B) \cap (A \cup C)) \subseteq (A \cup (B \cap C))$

B: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$