

CSC 240

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# PROOFS 1

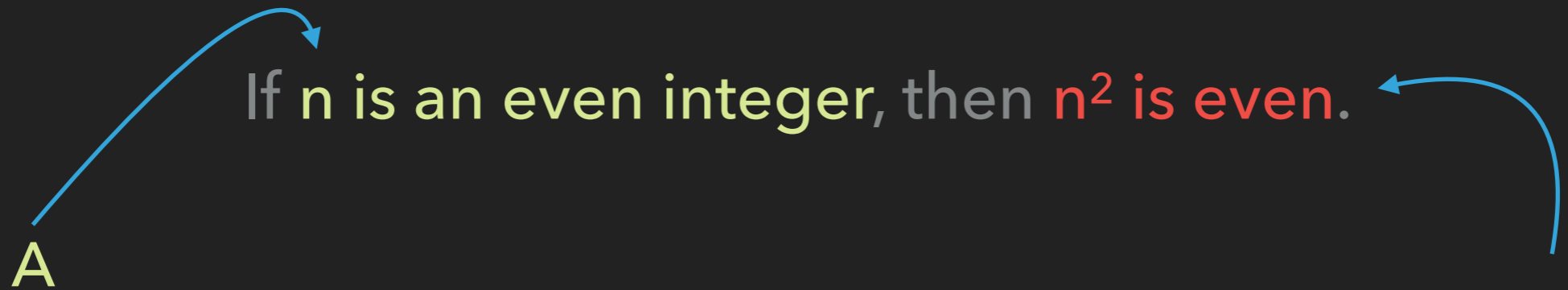
## DIRECT PROOFS - FORWARD BACKWARD METHOD

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- ▶ Ask a **key** question.
- ▶ Answer the key question **abstractly**.
- ▶ Transform the answer into **specifics** that match the problem.

# DIRECT PROOFS - FORWARD BACKWARD METHOD

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A:  $n$  is an even integer

$$A1: n = 2k$$

$$A2: n^2 = (2k)^2$$

$$= (2k)(2k)$$

$$= 4k^2$$

$$= 2(2k^2)$$

$$B1: n^2 = 2m$$

B:  $n^2$  is even

What it means to be "even"

Some algebra...

What it means to be "even"



What does it mean for a number to be even?

## DIRECT PROOFS - WRITING THE PROOF WITH WORDS

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**A:  $n$  is an even integer**

**A1:  $n = 2k$**

**A2:  $n^2 = (2k)^2$**   
 **$= (2k)(2k)$**   
 **$= 4k^2$**   
 **$= 2(2k^2)$**

**B1:  $n^2 = 2m$**

**B:  $n^2$  is even**

Let  $n$  be an even integer.

Since  $n$  is even, there's some integer  $k$  such that

$$n = 2k. \quad (1)$$

From (1) we see that

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2). \quad (2)$$

From (2) we see that there is an integer  $m$ , where  $m = 2k^2$  such that

$$n^2 = 2m. \quad (3)$$

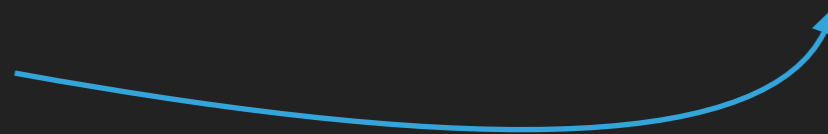
Therefore,  $n^2$  is even. ■

The end of proof marker.

Q.E.D.

"quod erat demonstrandum"

(what was to be demonstrated)



## DIRECT PROOFS - WRITING THE PROOF WITH LATEX

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\subsection{Theorem}
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If  $n$  is an even integer, then  $n^2$  is even.

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\subsection{Proof}
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Let  $n$  be an even integer. Since  $n$  is even, there's some integer  $k$  such that

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\begin{equation}
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\label{even1}
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$$n = 2k.$$

```
\end{equation}
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From [\ref{even1}](#), we see that

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\begin{equation}
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\label{even2}
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$$n^2 = (2k)^2 = 4k^2 = 2(2k^2).$$

```
\end{equation}
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From [\ref{even2}](#), we see that there is an integer  $m$  where  $m = 2k^2$  such that

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\begin{equation}
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\label{even3}
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$$n^2 = 2m.$$

```
\end{equation}
```

Therefore,  $n^2$  is even.  $\square$

# DIRECT PROOFS - FORWARD BACKWARD METHOD

If  $A$ ,  $B$ , and  $C$  are sets, then  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

A: Let  $x \in A \cup (B \cap C)$

A1: Then either  $x \in A$  or  $x \in (B \cap C)$

A2: Then either  $x \in A$  or ( $x \in B$  and  $x \in C$ )

Either...or = Proof by Cases

B2: For every element  $x \in A \cup (B \cap C)$ ,  
 $x \in (A \cup B) \cap (A \cup C)$

B1:  $(A \cup (B \cap C)) \subseteq ((A \cup B) \cap (A \cup C))$  and  
 $((A \cup B) \cap (A \cup C)) \subseteq (A \cup (B \cap C))$

B:  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

B

What "union" means

What "intersection" means

What it means for sets to be subsets of each other.

What it means for sets to be equal.

Choose either statement to be our starting point and work towards the other one

What does it mean a set to be a subset of another set?

What does it mean for two sets to be equal?

## DIRECT PROOFS - PROOF BY CASES / PROOF BY EXHAUSTION

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A2: Then either  $x \in A$  or  $(x \in B \text{ and } x \in C)$

Case 1:  $x \in A$

1.1: If  $x \in A$ , then  $x \in (A \cup B)$

What "union" means

1.2: If  $x \in A$ , then  $x \in (A \cup C)$

What "union" means

1.3: If  $x \in (A \cup B)$  and  $x \in (A \cup C)$ , then  
 $x \in (A \cup B) \cap (A \cup C)$

What "intersection" means

Case 2:  $x \in B$  and  $x \in C$

2.1: If  $x \in B$  then  $x \in (A \cup B)$

What "union" means

2.2: If  $x \in C$  then  $x \in (A \cup C)$

What "union" means

2.3: If  $x \in (A \cup B)$  and  $x \in (A \cup C)$ , then  
 $x \in (A \cup B) \cap (A \cup C)$

What "intersection" means

B2a: In every case,  $x \in (A \cup B) \cap (A \cup C)$

B2: For every element  $x \in A \cup (B \cap C)$ ,

$x \in (A \cup B) \cap (A \cup C)$

# DIRECT PROOFS - PROOF BY CASES / PROOF BY EXHAUSTION

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If  $A$ ,  $B$ , and  $C$  are sets, then  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

Does  $A \subseteq B$  necessarily mean  $B \subseteq A$ ?

A: Let  $x \in A \cup (B \cap C)$

C: Let  $x \in (A \cup B) \cap (A \cup C)$

A1: Then either  $x \in A$  or  $x \in (B \cap C)$

A2: Then either  $x \in A$  or ( $x \in B$  and  $x \in C$ )

?

A3: Proof by Cases 1 and 2

B2: For every element  $x \in A \cup (B \cap C)$ ,  
 $x \in (A \cup B) \cap (A \cup C)$

B2: For every element  $x \in (A \cup B) \cap (A \cup C)$ ,  
 $x \in A \cup (B \cap C)$

B1:  $(A \cup (B \cap C)) \subseteq ((A \cup B) \cap (A \cup C))$  and  
 $((A \cup B) \cap (A \cup C)) \subseteq (A \cup (B \cap C))$

B:  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$