CSC 240

FUNCTIONS

FIRST ORDER LOGIC

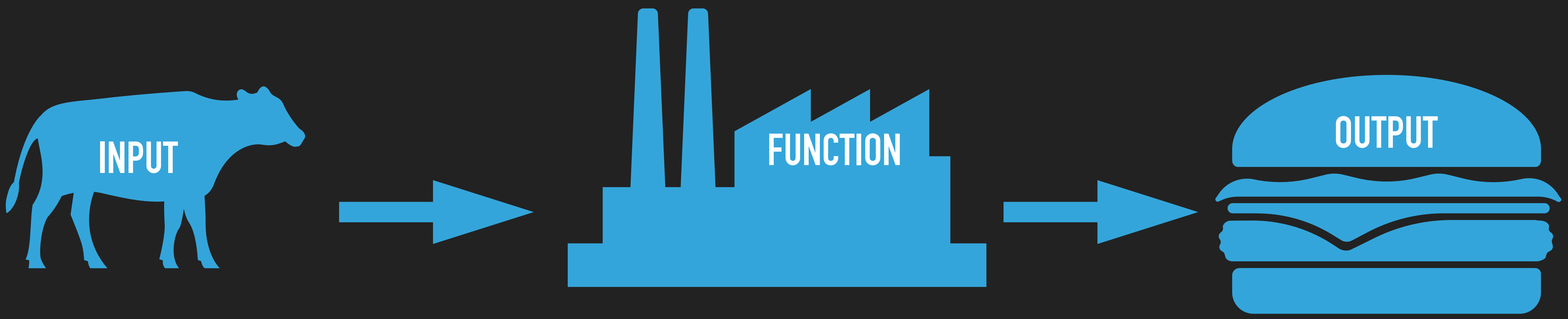
	Input	Output	Example
Connectives	Proposition(s)	A Proposition	$x \in A \rightarrow x \in A \cap B$
Predicates	Object(s)	A Proposition	IsFood(Carrot)
Functions	Object(s)	An Object	Breakfast(Yesterday)

FUNCTIONS

Functions: Produce on object based on the properties of another object.

SecretIdentity(Superman)   ColorOf(BrotherOf(Mario))

A function is an object that takes input and produces exactly one output.



Functions: Produce on object based on the properties of another object.

SecretIdentity(Superman)   ColorOf(BrotherOf(Mario))

A function is an object that takes input and produces exactly one output.

Math function: $f(x) = 2x + 3$

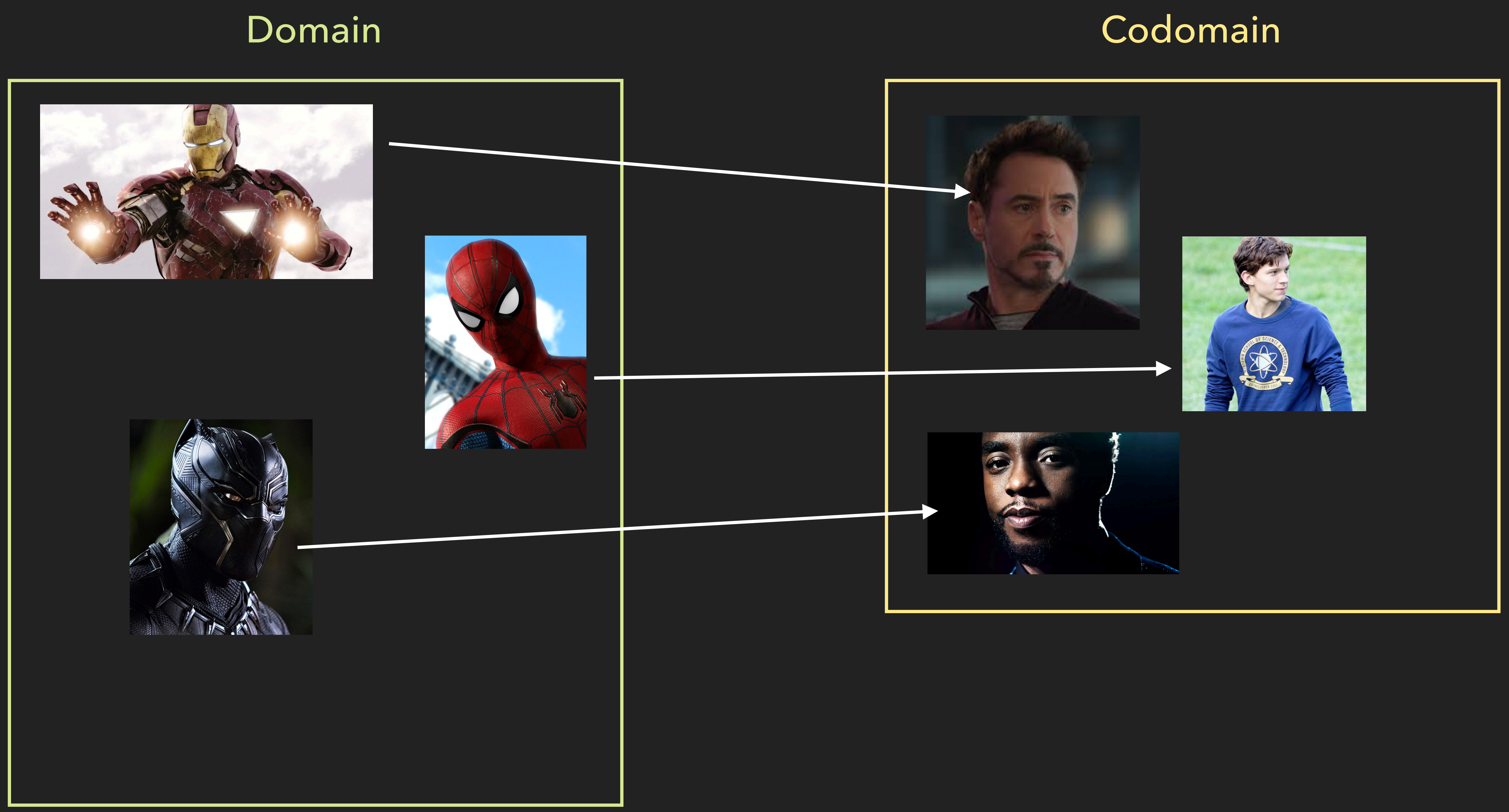
CS function:

```
int magnitude(Vector v) {
  return squareRoot(v.x * v.x + v.y * v.y);
}
```

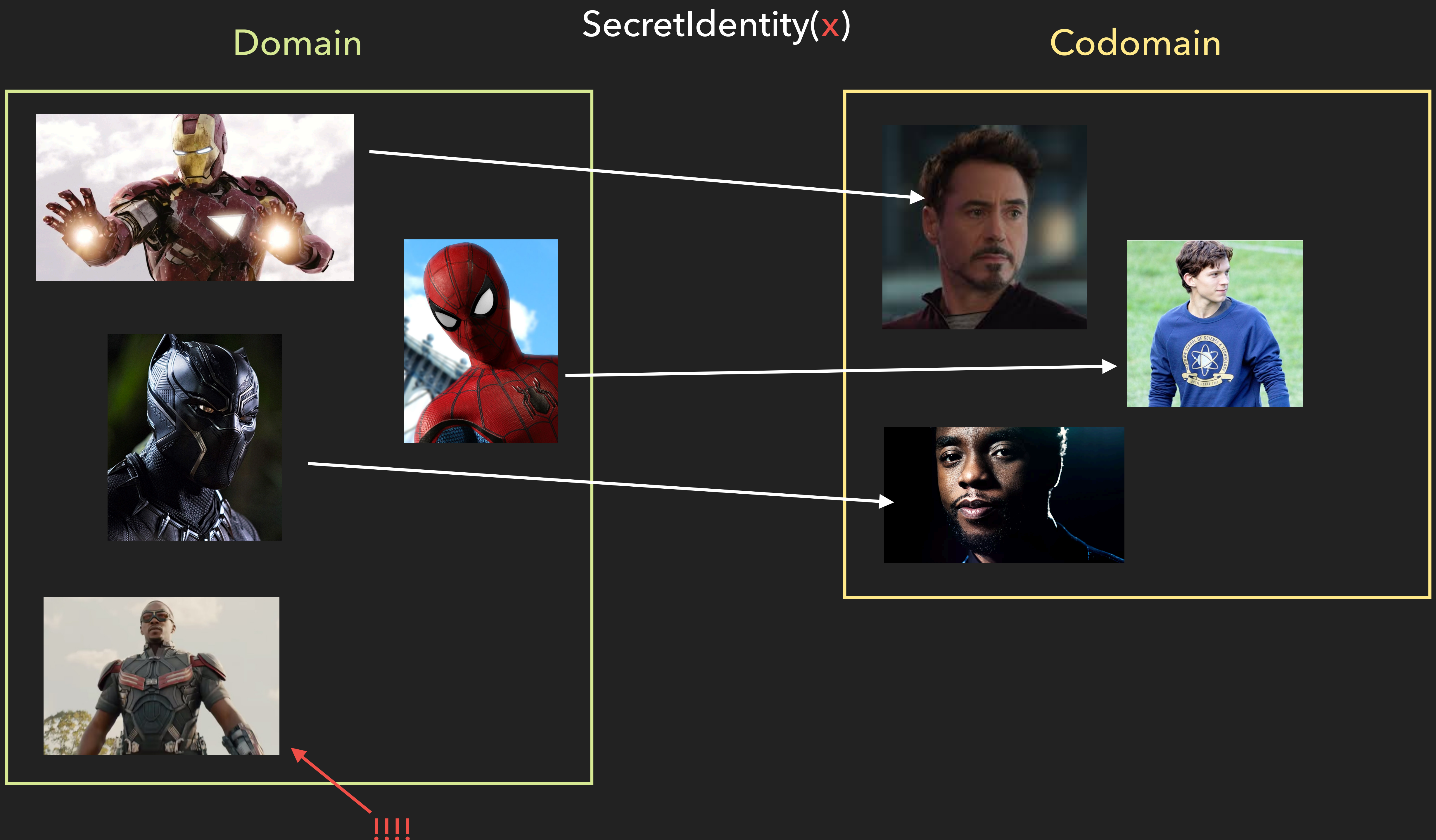
FUNCTIONS

A function maps an object from an element in its Domain to an element in its Codomain.

SecretIdentity(x)



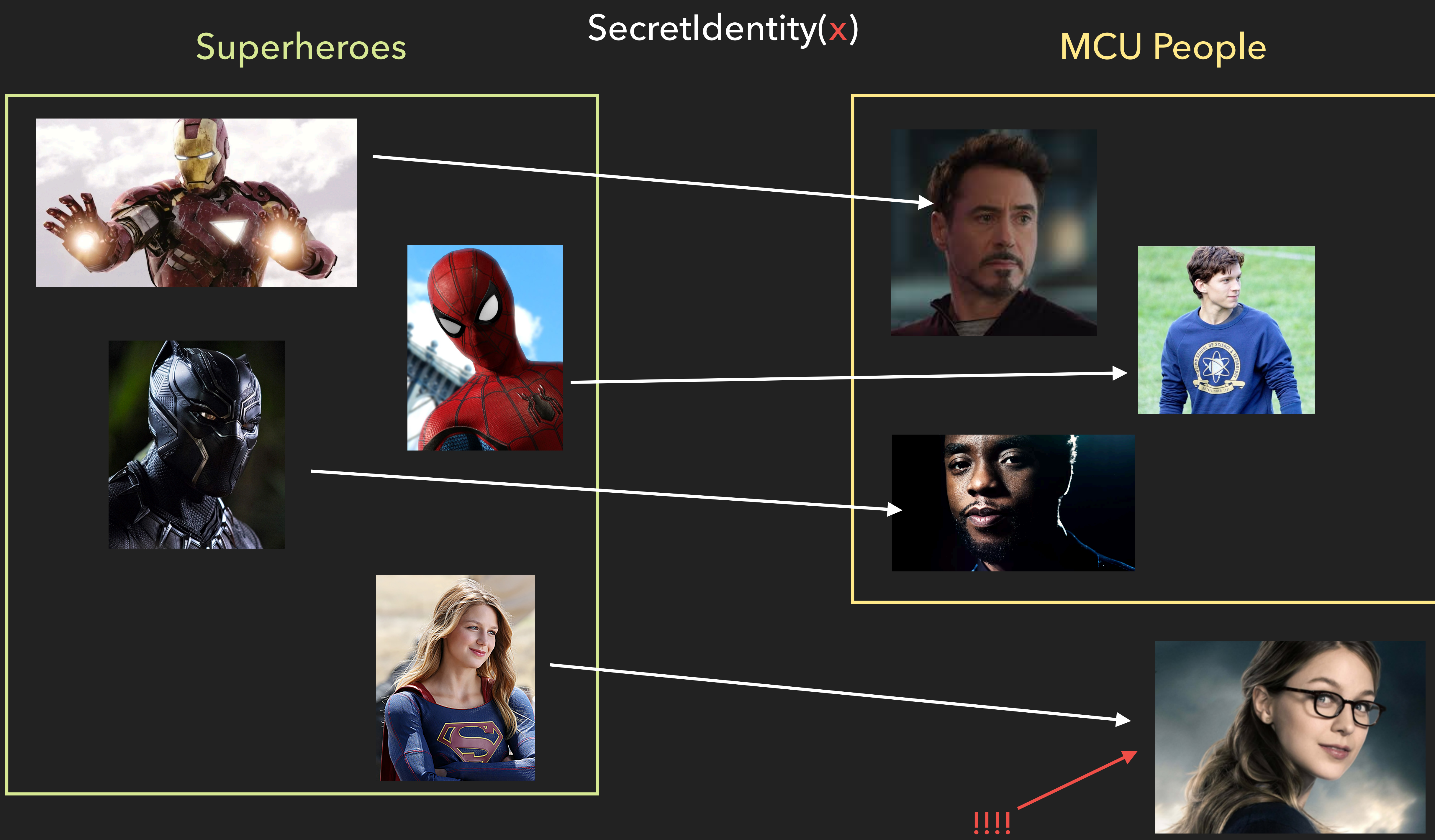
Rule 1: The function must produce an output for every element of the domain.



FUNCTIONS

Rule 1: The function must produce an output for every element of the domain.

Rule 2: The output of the function must be in its codomain.



Functions: Produce on object based on the properties of another object.

SecretIdentity(Superman)   ColorOf(BrotherOf(Mario))

A function is an object that takes input and produces exactly one output.

The domain of a function is the set of all possible inputs to that function.

The codomain of a function is the set of all possible outputs from that function.

Rule 1: The function must produce an output for every element of the domain.

Rule 2: The output of the function must be in its codomain.

The range of a function is the set of all actual outputs from that function.

If the domain of the function, f, is the set D and the codomain is the set C:

$$f : D \rightarrow C$$

$$\forall x \in D.\ \exists y \in C.\ f(x) = y$$

The function must produce an output for every element of the domain.

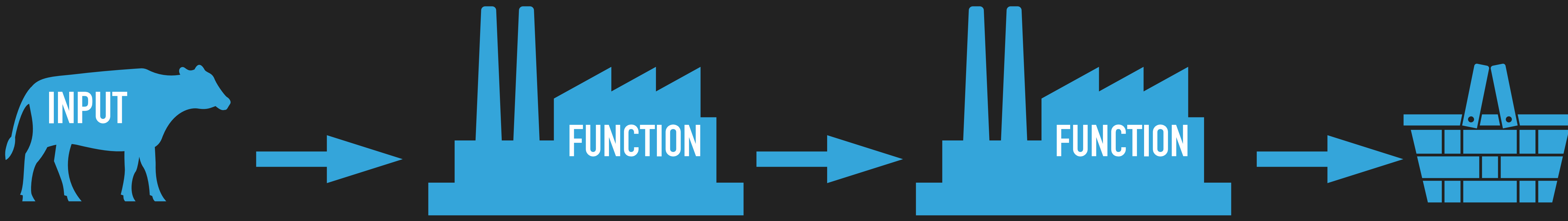
The output of the function must be in its codomain.

$$\forall x_1 \in D.\ \forall x_2 \in D.\ \left(x_1 = x_2 \rightarrow f(x_1) = (x_2)\ \right)$$

Functions must be deterministic.

FUNCTION COMPOSITION

functions can be combined



$$f : A \to B$$

$$g : B \to C$$

$$g \circ f : A \to C$$

$$(g \circ f)(x) : g(f(x))$$

Injective Functions:

$$\forall x_1 \in D.\ \forall x_2 \in D.\ (x_1 \neq x_2 \rightarrow f(x_1) \neq (x_2))$$

If the inputs are different, then the outputs will be different.

$$\forall x_1 \in D.\ \forall x_2 \in D.\ (f(x_1) = (x_2) \rightarrow x_1 = x_2)$$

If the outputs are the same, then the inputs were the same.

Which of these are injective?

$f(x) = x + 1$

$f(x) = x^2$

Surjective Functions:

$$\forall y \in C.\ \exists x \in D.\ \big(f(x) = y\ \big)$$

For every possible output, there is at least one input that produces it.

Which of these are surjective?

$f(x) = x$ where $f : \mathbb{Z} \rightarrow \mathbb{Z}$

$f(x) = x / 2$ where $f : \mathbb{Z} \rightarrow \mathbb{R}$

SPECIAL FUNCTIONS

Bijective Functions:

A function that is both injective and surjective.

Inverse Function:

A function f^{-1} is the inverse of function f if the following statements are true:

$$\forall x \in D. \left(f^{-1}(f(x)) = x \right) \qquad \forall y \in D. \left(f(f^{-1}(x)) = x \right)$$

A function with an inverse is called an invertible function.