CSC 240

LOGIC 3

How would you translate this statement?

Some cars fly in space.

$$\exists c.\ (\ \mathrm{Car}(c)\ \land\ \mathrm{FliesInSpace}(c)\ )$$

There exists an object (called c)

where it is true that

c is a car

and

c flies in space.

How would you translate this statement?

All dogs go to heaven.

$$\forall d.\ (\ \mathrm{Dog}(d) \rightarrow \mathrm{GoesToHeaven}(d)\ )$$

For every object, (called d)

it is true that

d being a dog

implies that

d goes to heaven.

Some A is a B.

$$\exists x.\,(\,A(x)\ \wedge\ B(x)\,)$$

All A's are B's

$$\forall x.\,(\,A(x)\ \rightarrow\ B(x)\,)$$

How would you translate this statement?

Real men don't eat quiche.

$$\forall m. \left(\mathrm{Man}(m) \rightarrow \neg \mathrm{EatsQuiche}(m) \right)$$

For every object, (called m)

it is true that

m being a man

implies that

m does not eat quiche.

How would you translate this statement?

Not all who wander are lost.

$$\exists w.\ (\ \mathrm{Wanders}(w)\ \land\ \neg\mathrm{Lost}(w)\ )$$

There exists an object, (called w)

for which it is true that

w wanders

and

w is not lost.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Some A is a B.

$\exists x. (A(x) \wedge B(x))$

All A's are B's

$\forall x. (A(x) \rightarrow B(x))$

No A is a B.

$\forall x. (A(x) \rightarrow \neg B(x))$

Some A's aren't B's

$\exists x. (A(x) \wedge \neg B(x))$

Remember:

We can prove any $\forall$… statement false with a single counter example.

We can prove any $\exists$… statement true with a single positive example.

How would you translate this statement?

Not all who wander are lost.

$$\exists w.\ (\ \text{Wanders}(w)\ \land \neg\text{Lost}(w)\ )$$

There exists an object, (called w)

for which it is true that

w wanders

and

w is not lost.

Given the following:

Person(p) - a predicate stating p is a person.

Loves(x, y) - a predicate stating that x loves y.

Translate this statement into first order logic:

Every person loves someone else.

For every object (called p)

it is true that

p being a person

implies

$$\forall p.\ (\ \mathrm{Person}(p)\ \rightarrow\ \exists q.\ (\ \mathrm{Person}(q)\ \wedge\ p \neq q\ \wedge\ \mathrm{Loves}(p, q)\ )\ )$$

that there exists an object (called q)

for which it is true that

q is a person

and p is not the same as q

and p loves q.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Given the following:

Person(p) - a predicate stating p is a person.

Loves(x, y) - a predicate stating that x loves y.

Translate this statement into first order logic:

There is a person that everyone else loves.

There exists an object (called p)

for which it is true that

p is a person

and

$$\exists p. \big(\text{Person}(p) \land \\ \forall q. \big(\text{Person}(q) \land p \neq q \to \text{Loves}(q, p) \big) \\ \big)$$

for every object (called q)

it is true that

q being a person

and p not being the same as q

implies that q loves p.

Quantifier Order Matters:

For any x, there's a y where P(x, y) is true

$$\forall x.\ \exists x.\ P(x, y)$$

There is an x, where for any y, P(x, y) is true.

$$\exists x.\ \forall x.\ P(x, y)$$

Given the following:

$Set(y)$ - a predicate stating that y is a set.

$x \in y$ - a predicate stating that x is an element of y.

Translate this statement into first order logic:

The empty set exists.

$$
\begin{aligned}
&\exists s.\ (\ Set(s)\ \land \\
&\quad \neg\exists x.\ (\ x \in s\ ) \\
&)
\end{aligned}
$$

$$
\begin{aligned}
&\exists s.\ (\ Set(s)\ \land \\
&\quad \forall x.\ (\ x \notin s\ ) \\
&)
\end{aligned}
$$

Given the following:

Set(y) - a predicate stating that y is a set.

$x \in y$ - a predicate stating that x is an element of y.

$x = y$ - a predicate stating that x is equal to y.

Translate this statement into first order logic:

Two sets are equal if and only if they contain the same elements.

$\forall s. \, (\mathrm{Set}(s) \rightarrow$ For all objects (called s), it is true that s being a set implies that…

$\forall T. \, (\mathrm{Set}(t) \rightarrow$ For all objects (called t), it is true that t being a set implies that…

$(s = t \leftrightarrow$ s is equal to t if and only if…

$\forall x. \, (x \in s \leftrightarrow x \in t)$

For all objects (called x), it is true that
x is an element of s if and only if x is an element of t.

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