

CSC 240

LOGIC 3

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

How would you translate this statement?

Some cars fly in space.

$\exists c. (\text{Car}(c) \wedge \text{FliesInSpace}(c))$

There exists an object (called c)

where it is true that

c is a car

and

c flies in space.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

How would you translate this statement?

All dogs go to heaven.

$\forall d. (\text{Dog}(d) \rightarrow \text{GoesToHeaven}(d))$

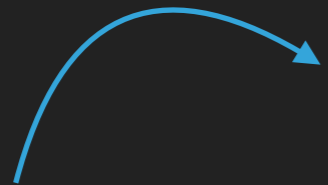
For every object, (called d)

it is true that

d being a dog

implies that

d goes to heaven.



TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Some A is a B.

$$\exists x. (A(x) \wedge B(x))$$

All A's are B's

$$\forall x. (A(x) \rightarrow B(x))$$

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

How would you translate this statement?

Real men don't eat quiche.

$\forall m. (\text{Man}(m) \rightarrow \neg \text{EatsQuiche}(m))$

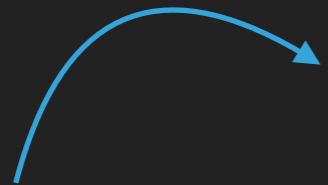
For every object, (called m)

it is true that

m being a man

implies that

m does not eat quiche.



TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

How would you translate this statement?

Not all who wander are lost.

$\exists w. (Wanders(w) \wedge \neg Lost(w))$

There exists an object, (called w)

for which it is true that

w wanders

and

w is not lost.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Some A is a B.

$$\exists x. (A(x) \wedge B(x))$$

All A's are B's

$$\forall x. (A(x) \rightarrow B(x))$$

No A is a B.

$$\forall x. (A(x) \rightarrow \neg B(x))$$

Some A's aren't B's

$$\exists x. (A(x) \wedge \neg B(x))$$

Remember:

We can prove any \forall ... statement **false** with a single counter example.

We can prove any \exists ... statement **true** with a single positive example.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

How would you translate this statement?

Not all who wander are lost.

$\exists w. (Wanders(w) \wedge \neg Lost(w))$

There exists an object, (called w)

for which it is true that

w wanders

and

w is not lost.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Given the following:

Person(p) - a predicate stating p is a person.

Loves(x, y) - a predicate stating that x loves y.

Translate this statement into first order logic:

Every person loves someone else.

For every object (called p)

it is true that

p being a person

$\forall p. (\text{Person}(p) \rightarrow$ implies

$\exists q. (\text{Person}(q) \wedge p \neq q \wedge \text{Loves}(p, q))$

)
that there exists
an object
(called q)

for which it
is true that

q is a person

and p is not the same as q

and p loves q.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Given the following:

Person(p) - a predicate stating p is a person.

Loves(x, y) - a predicate stating that x loves y.

Translate this statement into first order logic:

There is a person that everyone else loves.

There exists an object (called p)

for which it is true that

p is a person

$\exists p. (\text{Person}(p) \wedge$

and

$\forall q. (\text{Person}(q) \wedge p \neq q \rightarrow \text{Loves}(q, p))$

)

for every object
(called q)

it is true that

q being a person

and p not being the same as q

implies that q loves p.

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Quantifier Order Matters:

For any x , there's a y where $P(x, y)$ is true

$$\forall x. \exists y. P(x, y)$$

There is an x , where for any y , $P(x, y)$ is true.

$$\exists x. \forall y. P(x, y)$$

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Given the following:

$\text{Set}(y)$ - a predicate stating that y is a set.

$x \in y$ - a predicate stating that x is an element of y .

Translate this statement into first order logic:

The empty set exists.

$$\begin{aligned} & \exists s. (\text{Set}(s) \wedge \\ & \quad \neg \exists x. (x \in s) \\ &) \end{aligned}$$

$$\begin{aligned} & \exists s. (\text{Set}(s) \wedge \\ & \quad \forall x. (x \notin s) \\ &) \end{aligned}$$

TRANSLATING STATEMENTS INTO FIRST ORDER LOGIC

Given the following:

$\text{Set}(y)$ - a predicate stating that y is a set.

$x \in y$ - a predicate stating that x is an element of y .

$x = y$ - a predicate stating that x is equal to y .

Translate this statement into first order logic:

Two sets are equal if and only if they contain the same elements.

$\forall s. (\text{Set}(s) \rightarrow$ For all objects (called s), it is true that s being a set implies that...

$\forall t. (\text{Set}(t) \rightarrow$ For all objects (called t), it is true that t being a set implies that...

$(s = t \leftrightarrow$ s is equal to t if and only if...

$\forall x. (x \in s \leftrightarrow x \in t)$

For all objects (called x), it is true that
 x is an element of s if and only if x is an element of t .

)

)

)