CSC 240

DECIDABILITY

UN

A Turing machine accepts a string if it enters an accept state while processing that string.

A Turing machine rejects a string if it enters a reject state while processing that string.

A Turing machine loops forever on a string if it never enters an accept or reject state while processing that string.

A Turing machine halts on a string if it accepts or rejects that string.

A language is "Turing Recognizable" if a Turing machine recognizes it.

A language is "Turing Decidable" if a Turing machine decides it, that is if the strings of that language cause the Turing machine to end up in an accept or reject state.

Can we build a Turing Machine that *recognizes* A_{TM}?

Can we build a Turing Machine that *decides* A_{TM}?

ACCEPTANCE PROBLEMS

We can build a Turing Machine that tells us:

If a DFA will accept a string.

If an NFA will accept a string.

If a Regular Expression will accept a string.

If a CFG describes a string.

If a PDA accepts a string.

M - Some Turing Machine

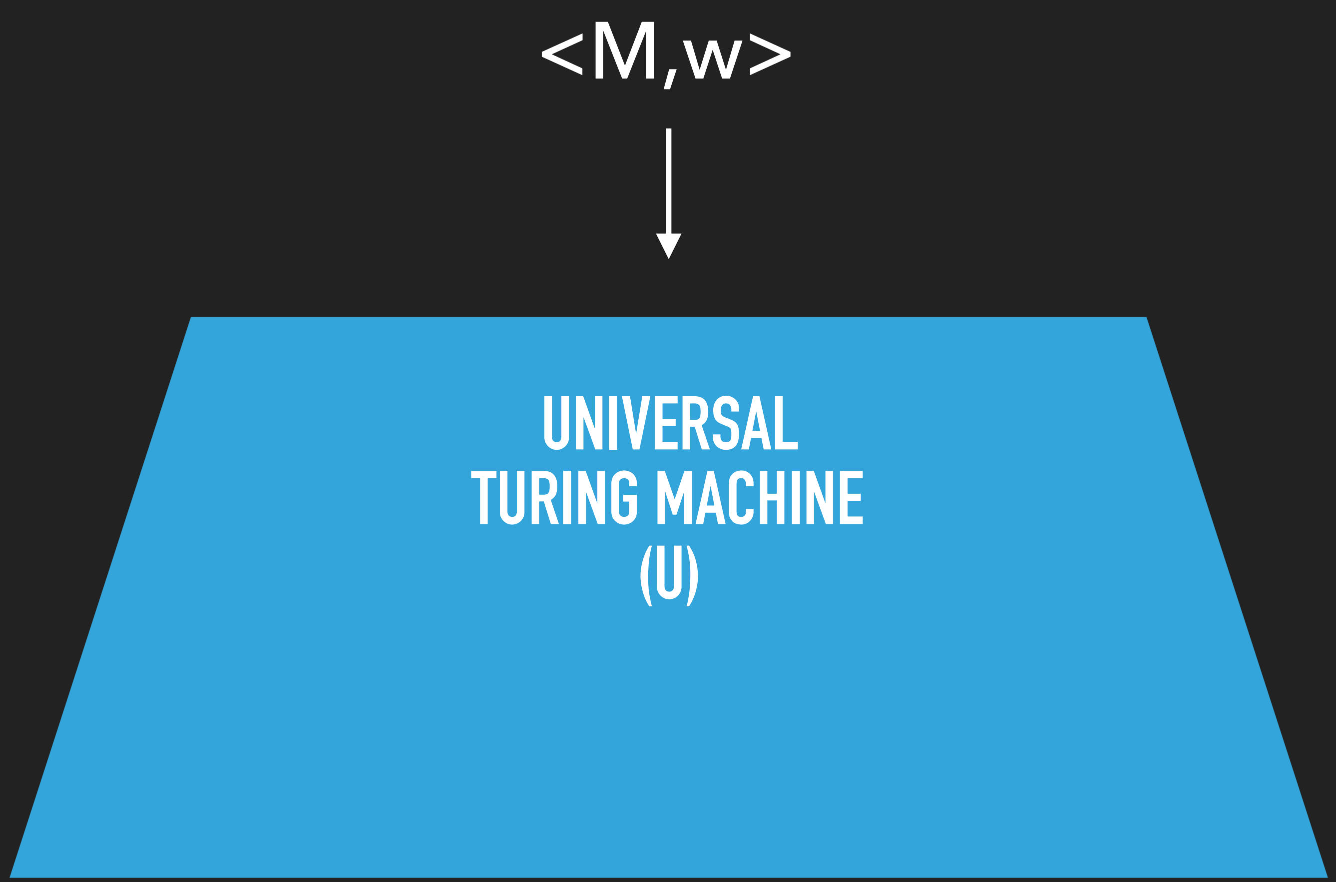
<M, w> - Some Turing Machine and its string input encoded as a string

A_{TM} = { <M, w> | M is a TM that accepts string w }

Can we build a Turing Machine that *decides* A_{TM}?

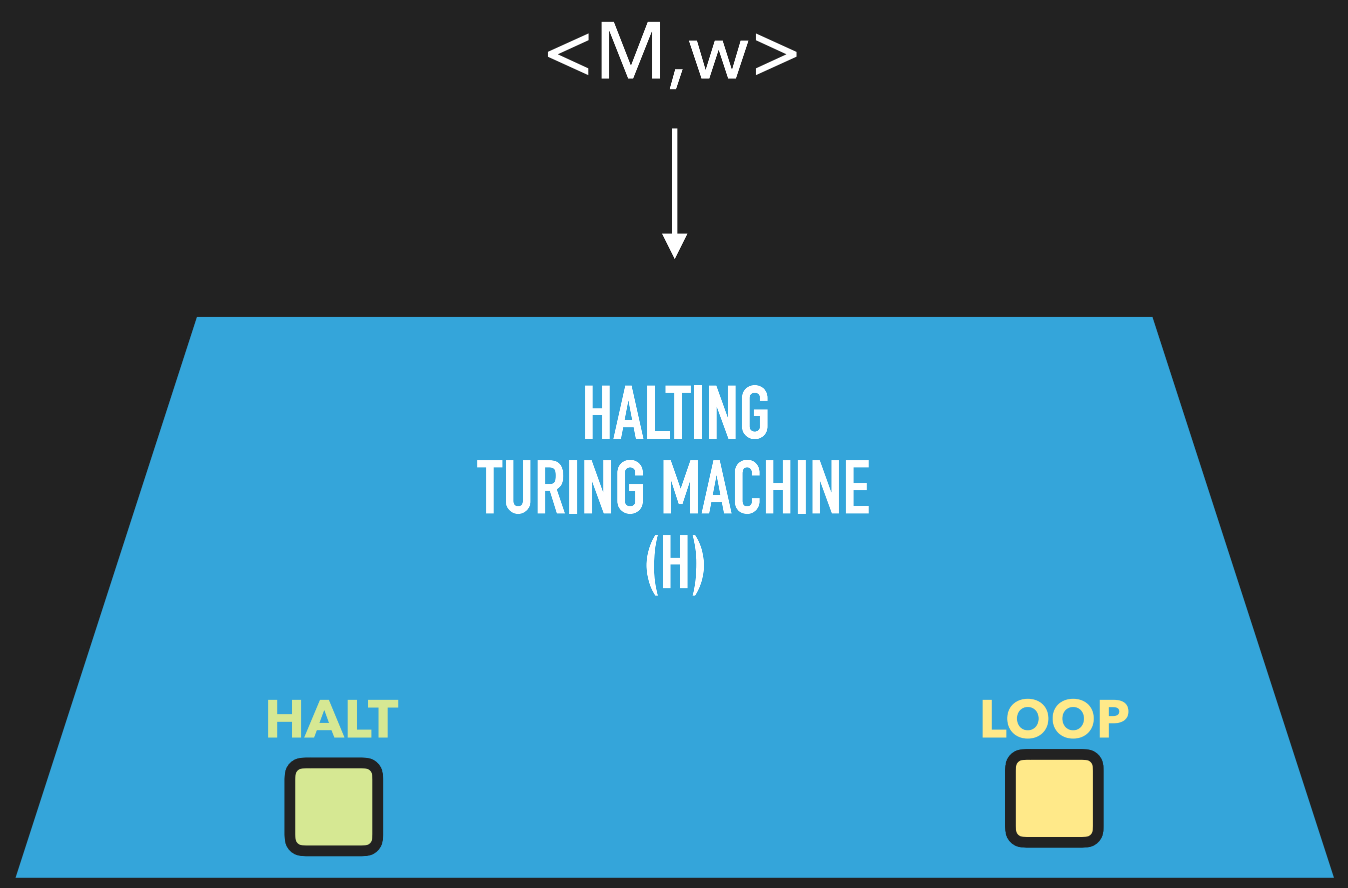
In other words, can we build a Turing Machine that tells us, for an arbitrary TM, M and an arbitrary input string w, if M will accept or reject w?

UNIVERSAL TURING MACHINE



U is a Universal Turing Machine

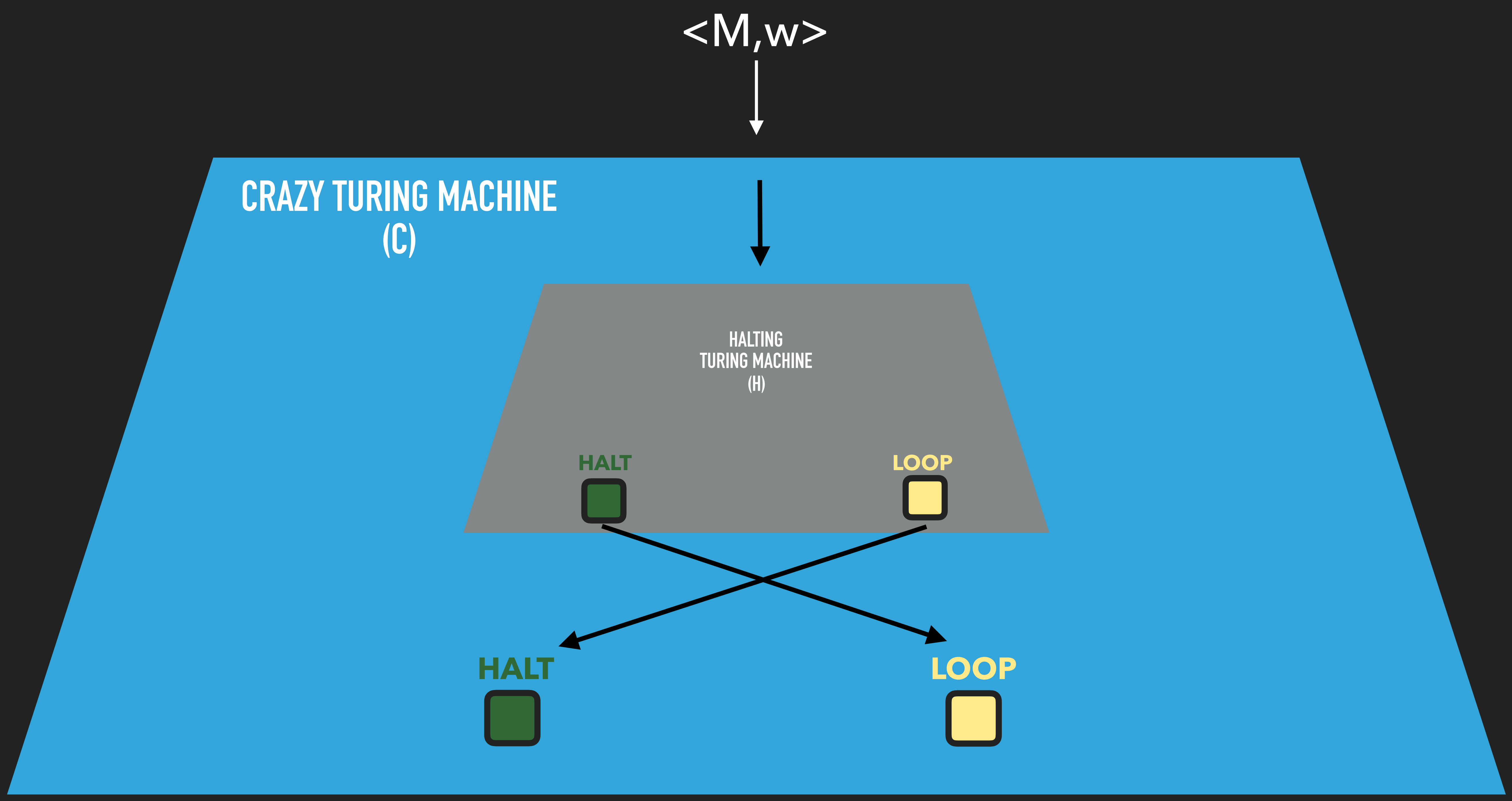
It can accept any Turing Machine and an input string and run the Turing Machine as it processes w.



If M halts on w, H will halt on <M,w>

If M loops on w, H will loop on <M,w>

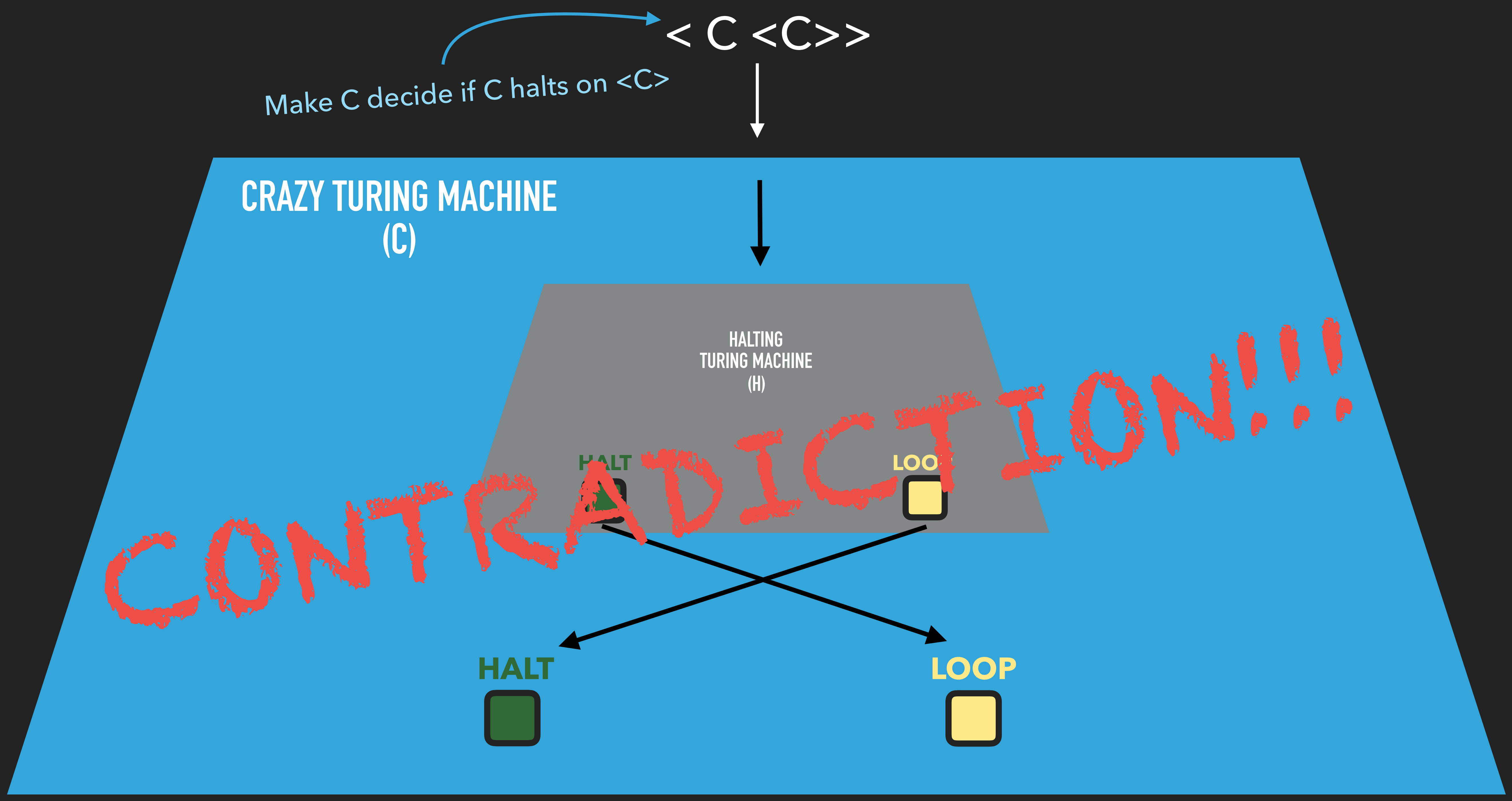
THE HALTING MACHINE



If M halts on w, C will loop on <M,w>

If M loops on w, C will halt on <M,w>

THE HALTING MACHINE



If C halts on <C>, C will loop on <C>

If C loops on <C>, C will halt on <C>

ACCEPTANCE PROBLEMS

<M, w> - Some Turing Machine and its string input encoded as a string

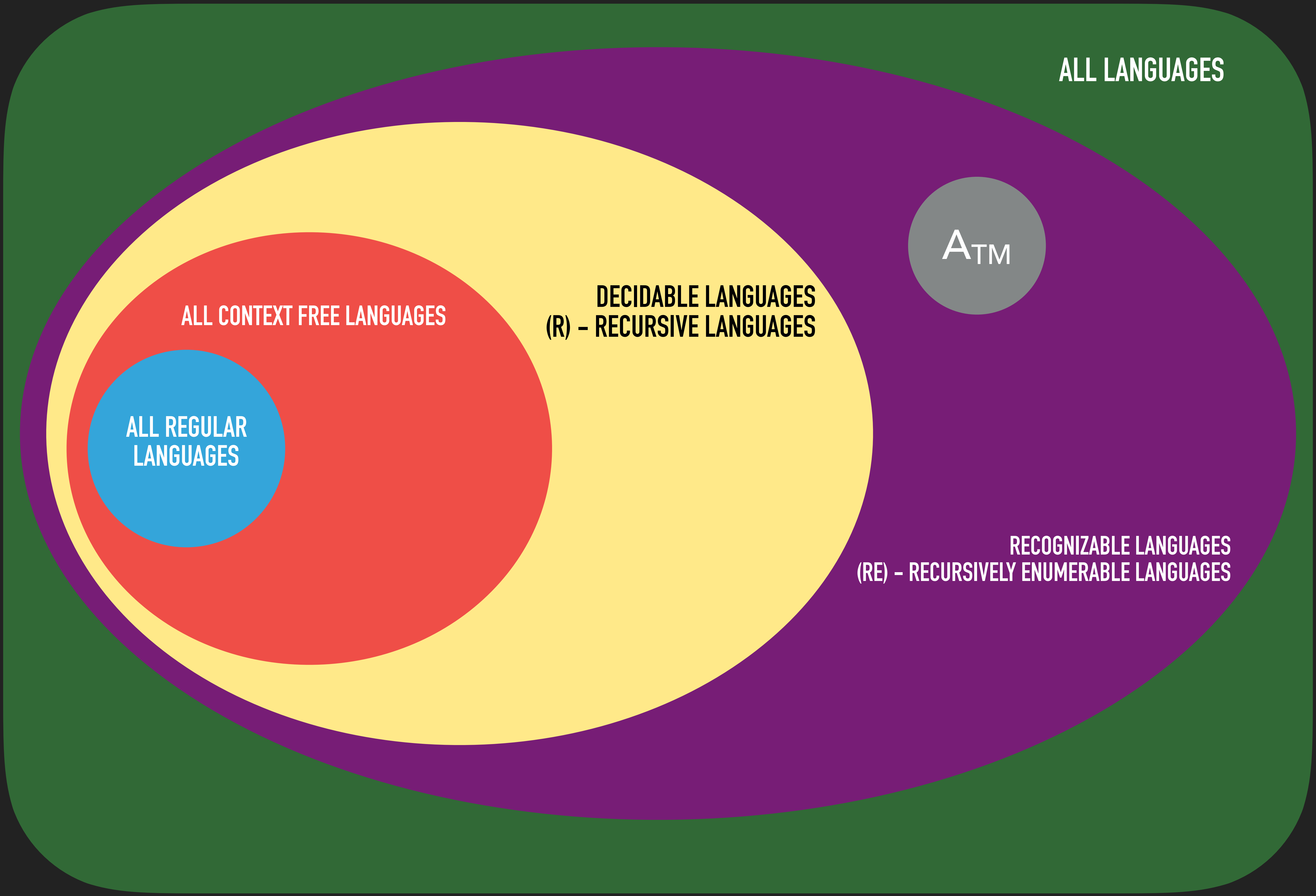
A_{TM} = { <M, w> | M is a TM that accepts string w }

Can we build a Turing Machine that *decides* A_{TM}?

In other words, can we build a Turing Machine that tells us, for an arbitrary TM, M and an arbitrary input string w, if M will accept or reject w?

NO! A_{TM} is recognizable, but not decidable.

TURING DECIDABLE LANGUAGES



ACCEPTANCE PROBLEMS

We can build a Turing Machine that tells us:

- If a DFA will accept a string.
- If an NFA will accept a string.
- If a Regular Expression will accept a string.
- If a CFG describes a string.
- If a PDA accepts a string.

} Decidable

We can't build a Turing Machine that tells us:

- If a TM will halt (accept or reject) on a string. — Not Decidable

All of these problems (languages) are *recognizable* by at least one TM.

Are there problems (languages) *not recognizable* by any TM?

$$\mathbf{TM} = \{M_0, M_1, M_2, M_3, \ldots\}$$

M_0 recognizes these strings…

	<M_0>	<M_1>	<M_2>	<M_3>	…
M_0	Y	N	Y	Y	…
M_1	N	Y	Y	Y	…
M_2	Y	Y	N	N	…
M_3	N	N	Y	N	…
…	…	…	…	…	…

$$TM = \{M_0, M_1, M_2, M_3, \ldots\}$$

	$<M_0>$	$<M_1>$	$<M_2>$	$<M_3>$	…
M_0	**Y**	**N**	**Y**	**Y**	**…**
M_1	**N**	**Y**	**Y**	**Y**	**…**
M_2	**Y**	**Y**	**N**	**N**	**…**
M_3	**N**	**N**	**Y**	**N**	**…**
…	**…**	**…**	**…**	**…**	**…**
L_D	**Y**	**Y**	**N**	**N**	**…**

The "Diagonal Language"

does this row have a pairing?

UNRECOGNIZABLE LANGUAGES

$$TM = \{M_0, M_1, M_2, M_3, \ldots\}$$

	$<M_0>$	$<M_1>$	$<M_2>$	$<M_3>$	...
M_0	**Y**	**N**	**Y**	**Y**	...
M_1	**N**	**Y**	**Y**	**Y**	...
M_2	**Y**	**Y**	**N**	**N**	...
M_3	**N**	**N**	**Y**	**N**	...
...	...	...	...	...	...
$\overline{L_D}$	**N**	**N**	**Y**	**Y**	...

Generate the complement of the row by flipping its Y's and N's.

does this row have a pairing?

The Complement of the Diagonal Language.

UNRECOGNIZABLE LANGUAGES

$$TM = \{M_0, M_1, M_2, M_3, \dots\}$$

N	N	Y	Y	...

$$M_? \longleftrightarrow \{ \langle M_3 \rangle, \langle M_4 \rangle, \dots \}$$

This language is not recognizable by any Turing Machine!

There is NO Turing Machine that will recognize this set of strings.

$\overline{L_D}$ is not recognizable by any Turing Machine

$$|TM| < |\mathscr{L}|$$

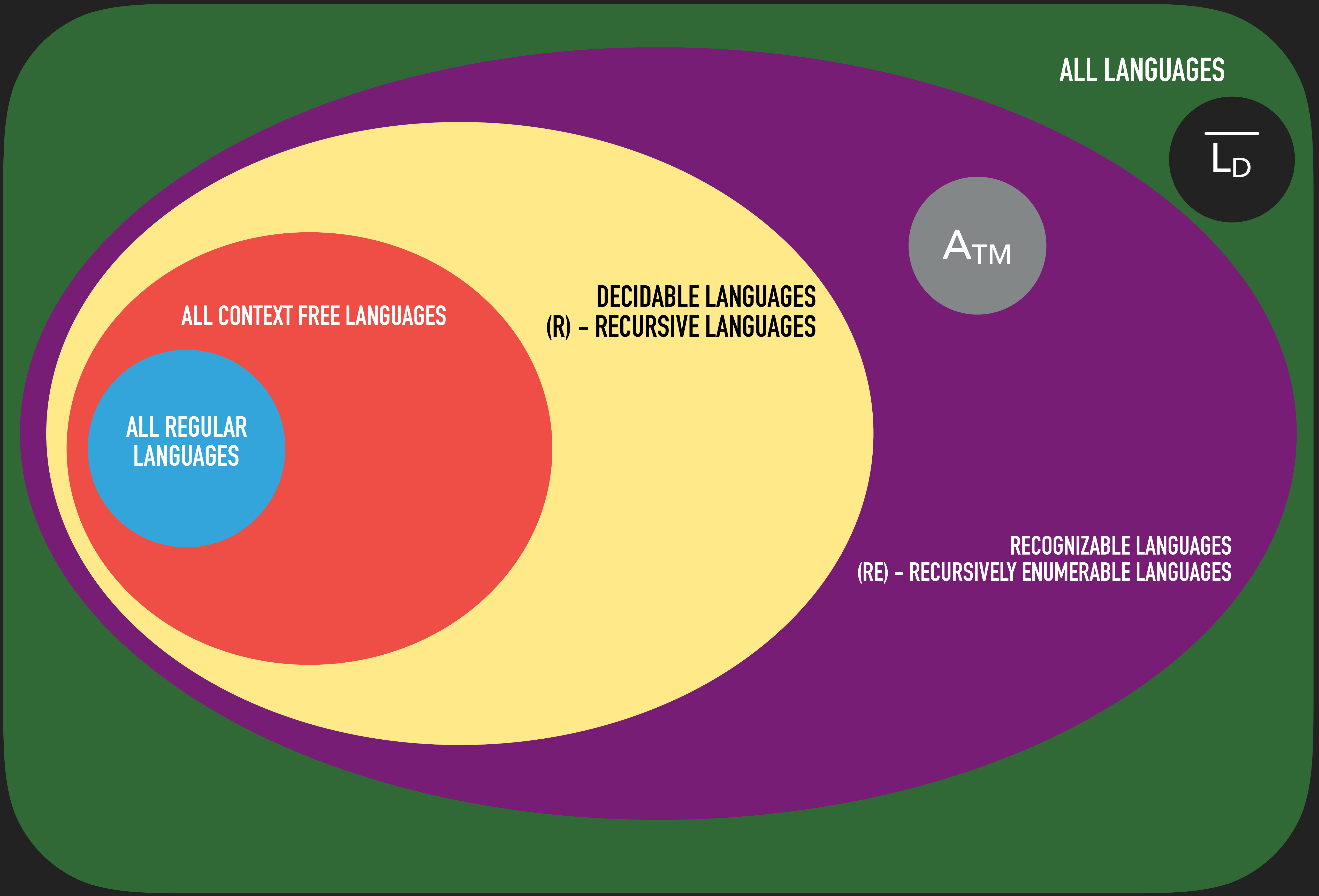
How many different possible Turing machines are there?

How many different possible Languages are there?

$\overline{L_D}$ is not recognizable by any Turing Machine

$$|TM| < |\mathscr{L}|$$

LANGUAGES


ALL LANGUAGES
$\overline{L_D}$
A_{TM}
DECIDABLE LANGUAGES
(R) – RECURSIVE LANGUAGES
ALL CONTEXT FREE LANGUAGES
ALL REGULAR LANGUAGES
RECOGNIZABLE LANGUAGES
(RE) – RECURSIVELY ENUMERABLE LANGUAGES