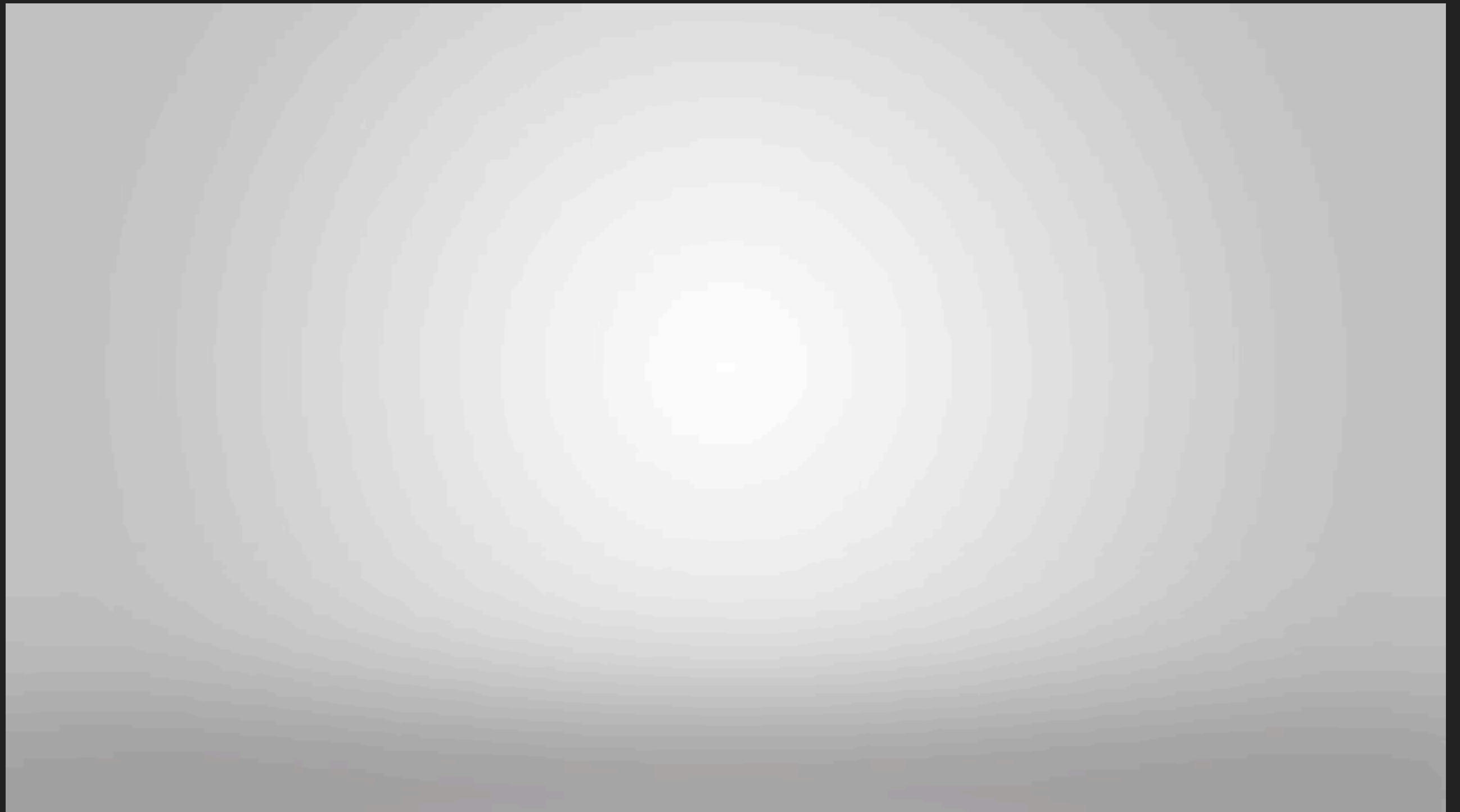


CSC 240

TURING MACHINES

WHAT IS A TURING MACHINE?



TURING MACHINES VS FINITE AUTOMATA

Turing machines can read from the input tape and write to it.

The read/write head can move left and right.

The input tape is infinite in both directions.

Special **Accept** and **Reject** states occur immediately.

TURING MACHINE

Defined by a 7-tuple: $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$

Q : A finite set called **states**.

Σ : A finite set called the **input alphabet**.

Γ : A finite set called the **output alphabet**.

$\delta: Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$ is the **transition function**.

q_0 : is the **start state** where $q_0 \in Q$.

q_{accept} : is the **accept state**

q_{reject} : is the **reject state**, where $q_{\text{reject}} \neq q_{\text{accept}}$

Transitions:

$a \rightarrow b, D$

read input a , write b to the tape, and move the read/write head in direction D

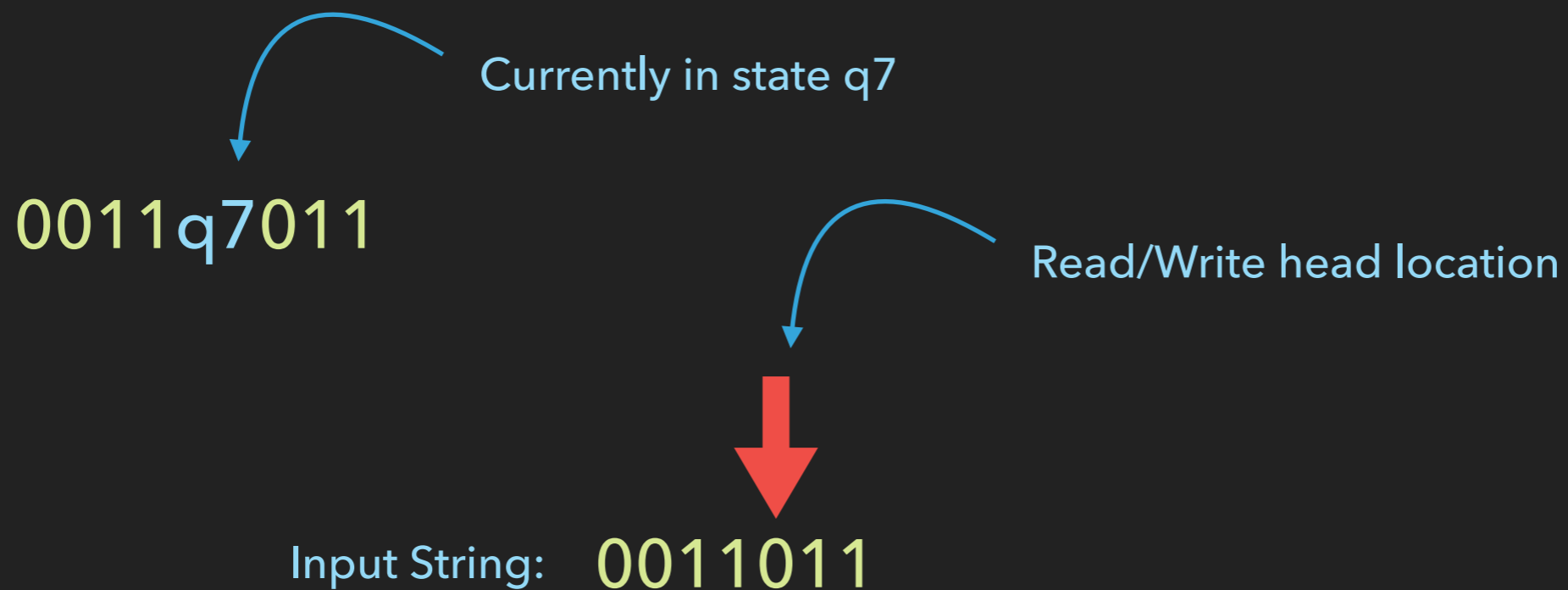
TURING MACHINE CONFIGURATIONS

uqv

q : The current state

uv : The input string

Read head positioned over the first symbol of v .



TURING MACHINE CONFIGURATIONS

We can chart the progress of a Turing machine through a series of configurations:

C_0 : $q_00011011$

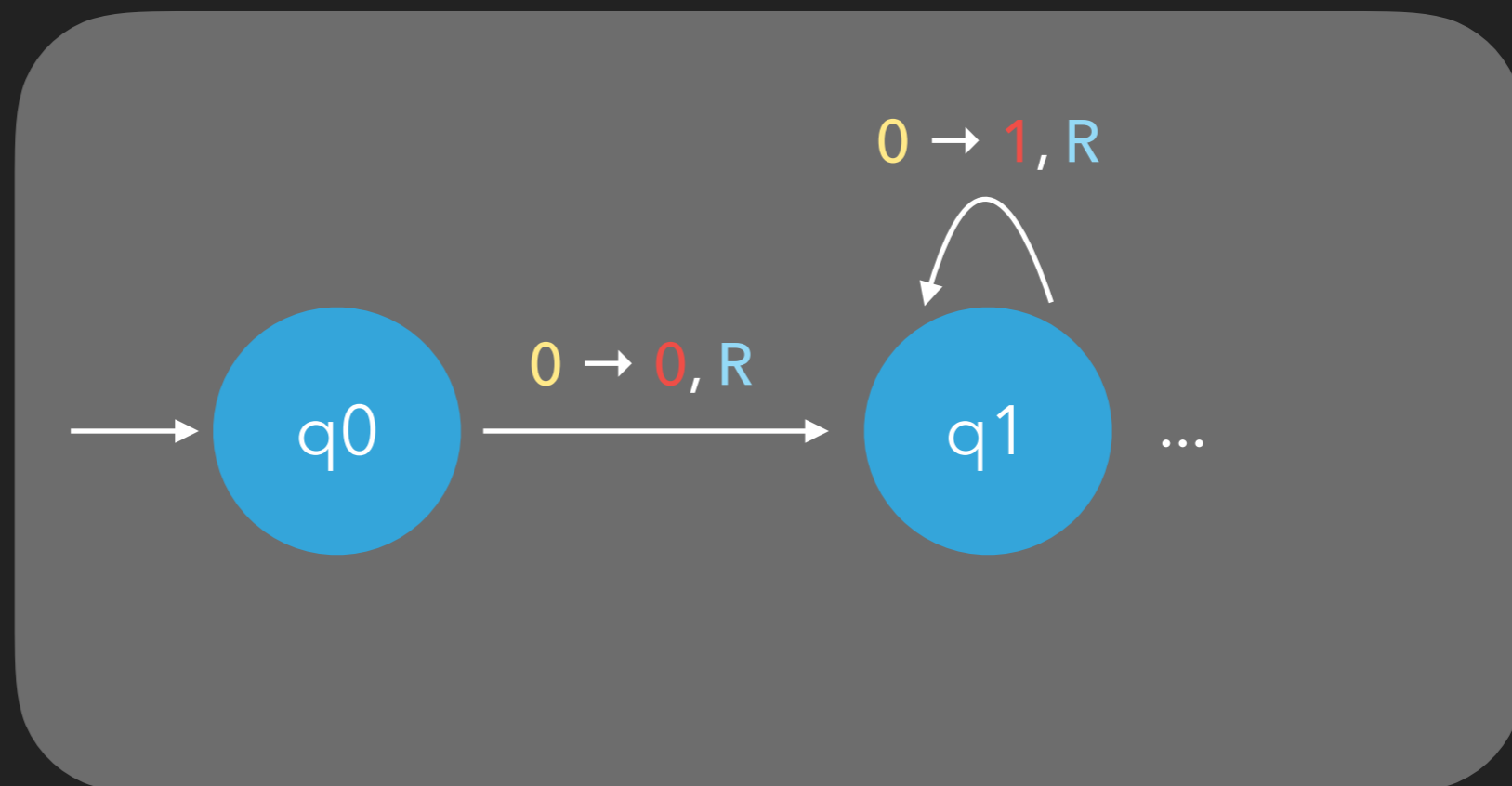
C_1 : $0q_1011011$

C_2 : $01q_111011$

C_0 *yields* C_1

C_1 *yields* C_2

etc...



TURING LANGUAGES

A Turing machine **accepts** a string if it enters an accept state while processing that string.

A Turing machine **rejects** a string if it enters a reject state while processing that string.

A Turing machine **loops** forever on a string if it never enters an accept or reject state while processing that string.

A Turing machine **halts** on a string if it accepts or rejects that string.

TURING LANGUAGES

The set of strings accepted by a Turing Machine are the language of that machine.

$$\mathcal{L}(M) = A$$

The *Language* of Turing machine M

A is the set of all strings accepted by M.


"A is the language of M"

"A is the *recognized by* M"

TURING LANGUAGES

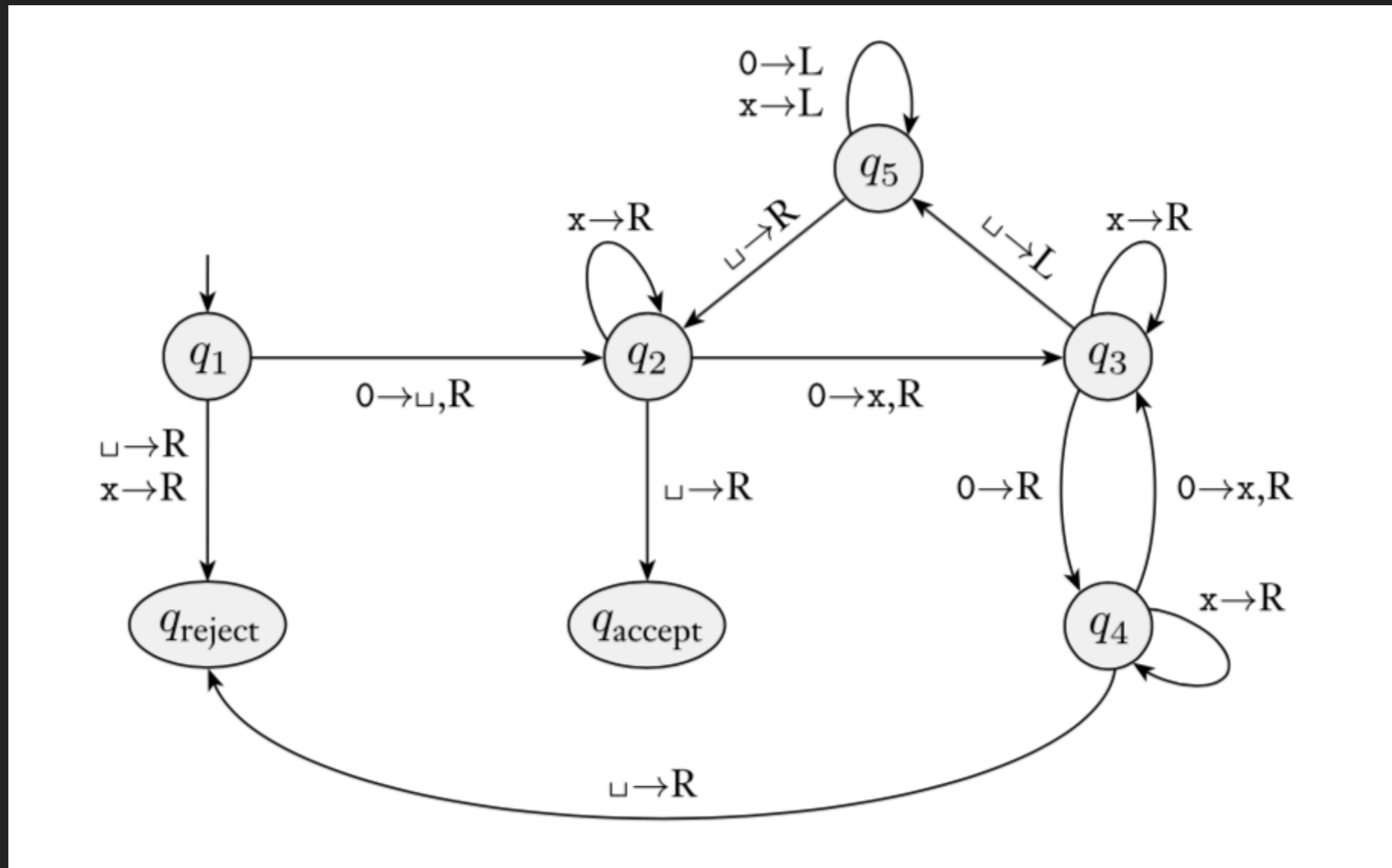
A language is “**Turing Recognizable**” if a Turing machine recognizes it.

A language is “**Turing Decidable**” if a Turing machine decides it, that is if the strings of that language cause the Turing machine to end up in an **accept** or **reject** state.



These machines are sometimes called “deciders”

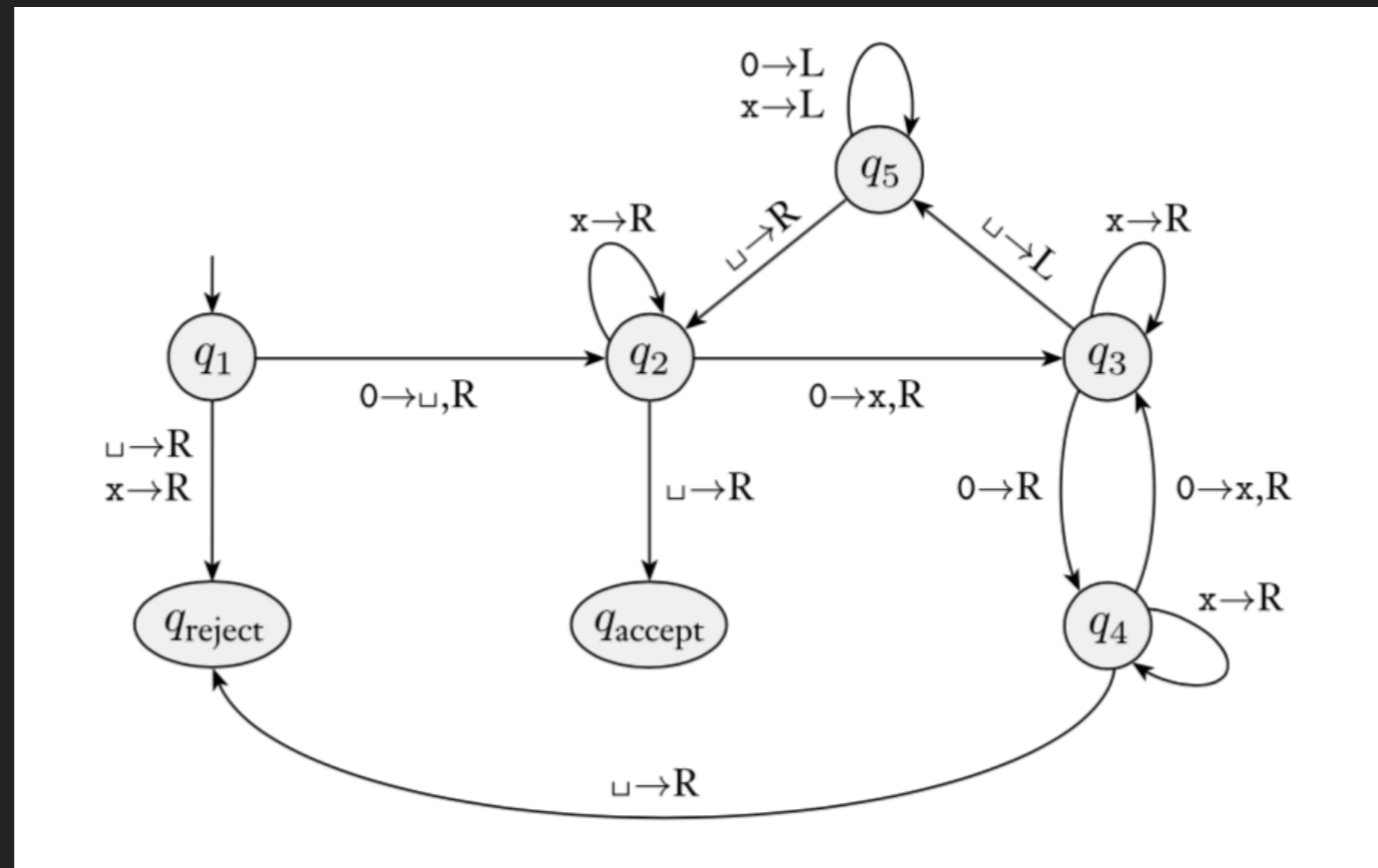
TURING MACHINE



$$A = \{0^{2^n} \mid n \geq 0\}$$

Strings with an even number of 0's

TURING MACHINE



On input string w :

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0, accept.
3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, reject.
4. Return the head to the left-hand end of the tape.
5. Go to stage 1.