CSC 240

PUSHDOWN AUTOMATA

WAYS TO DESCRIBE A LANGUAGE

Finite automata, which recognize strings in the language.

Regular expressions, which describe strings in the language.

Context-Free grammar, which describe structure of the language.

RLS ARE CFLS BUT CFLS ARE NOT ALL RLS

$$L = \{ a^n b^n \mid n \in \mathbb{N} \}$$

Can we build an NFA (or DFA or Regular Expression) for this language?

No!

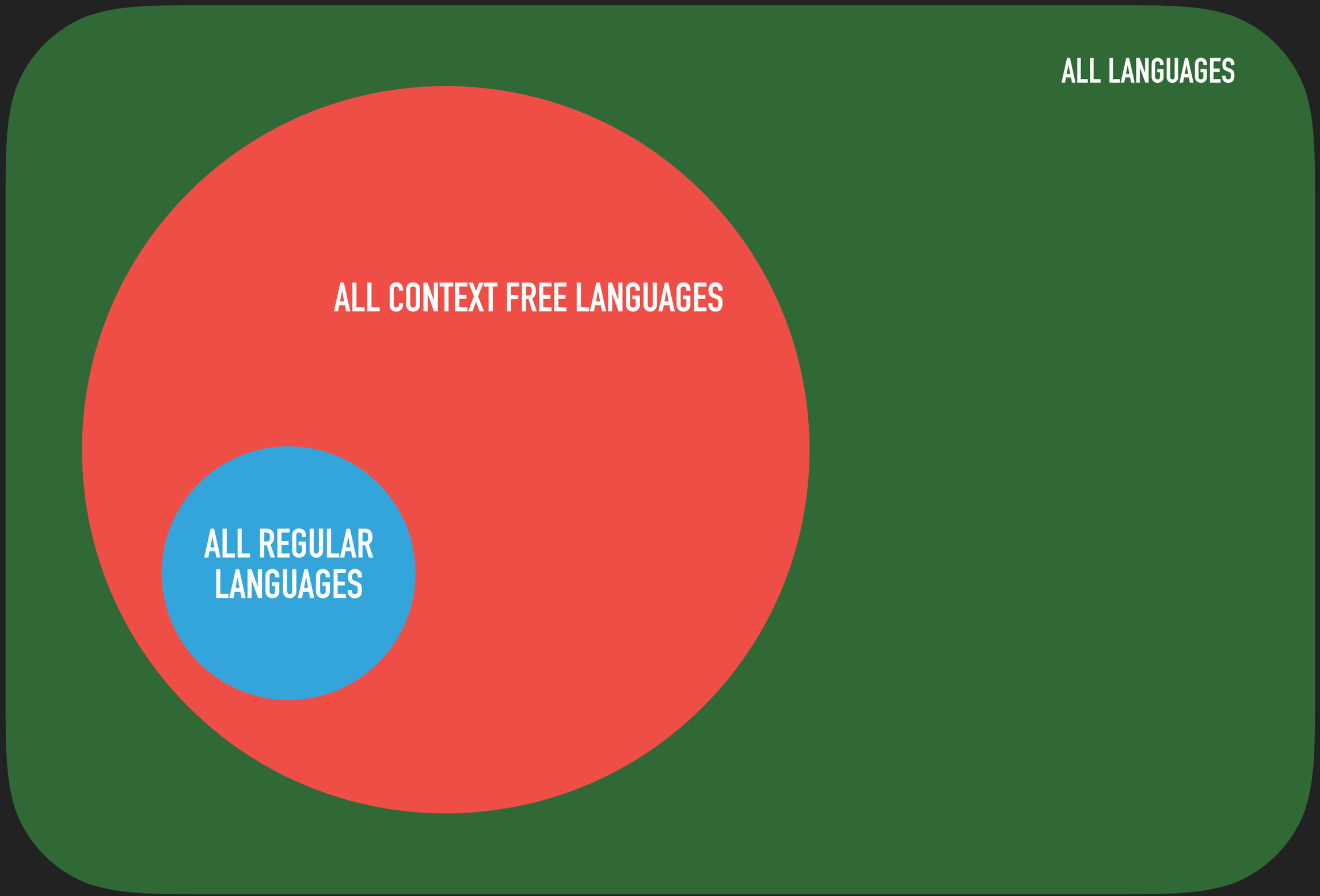
This language requires an infinite amount of memory and no FINITE automata has INFINITE memory.

Can we build a CFG for this language?

Yes!

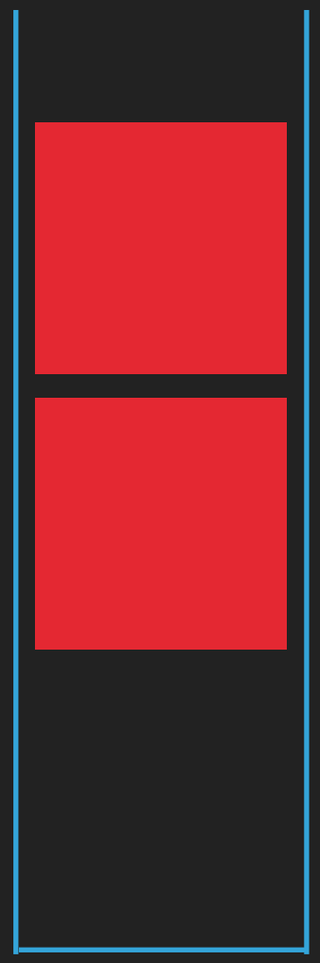
Because of their recursive nature, a CFG allows us to describe languages that require INFINITE memory.

RLS ARE CFLS BUT CFLS ARE NOT ALL RLS



STACKS

2. Pop Items off of the Stack

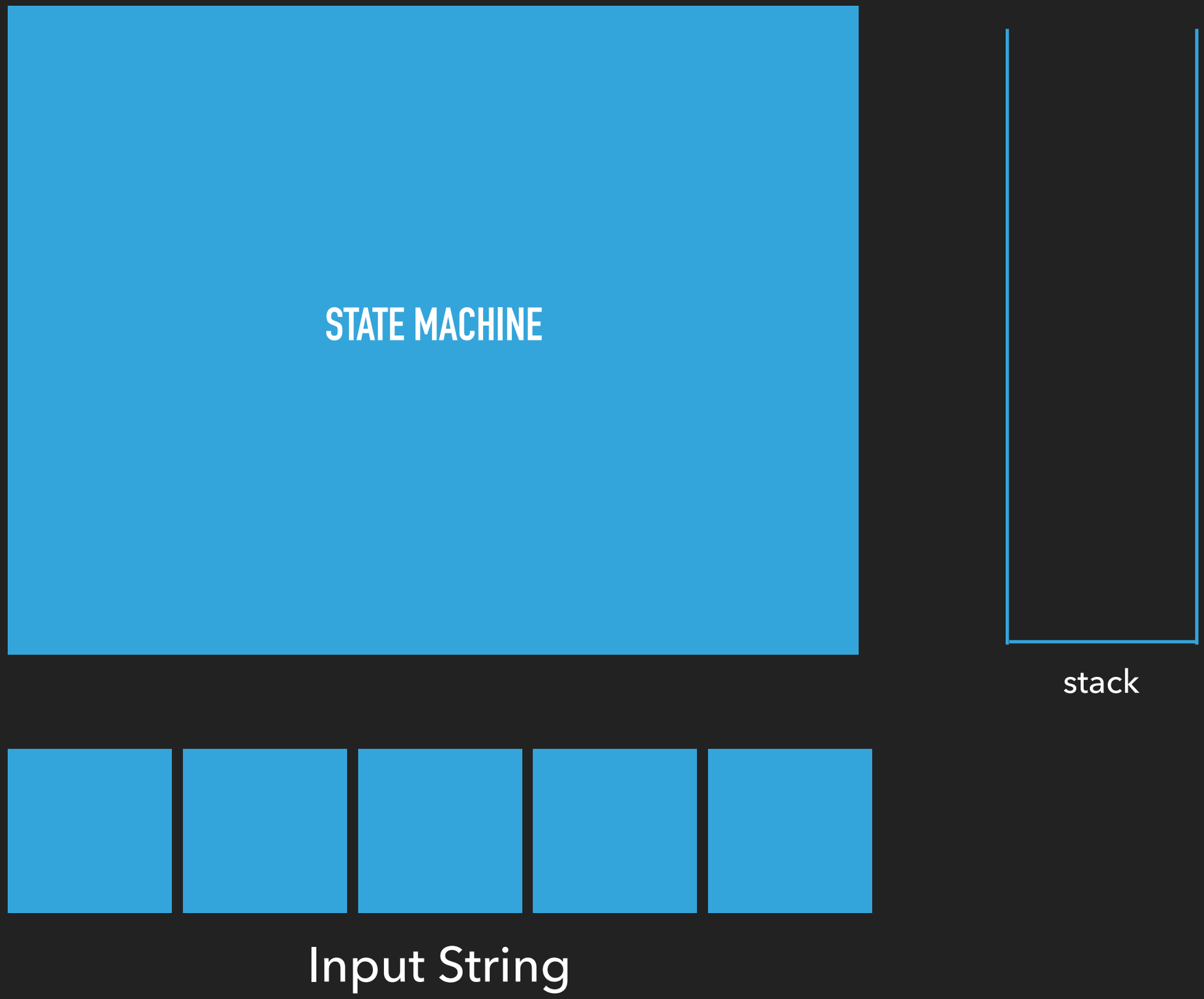


stack

1. Push Items onto the Stack

We can only push to or pop from the top of the stack.

PUSHDOWN AUTOMATA



PUSHDOWN AUTOMATA

PDA: Defined by a 6-tuple: $(Q, \Sigma, \Gamma, \delta, q_0, F)$

Q: A finite set called states.

Σ: A finite set called the input alphabet.

Γ: A finite set called the stack alphabet.

δ: $Q \times \Sigma \rightarrow Q$ is the transition function.

q_0: is the start state where $q0 \in Q$.

F: is the set of accepting states where $F \subseteq Q$.

Transitions:

a, b → c

read input a, pop b off the stack, and push c onto the stack

only take the transition if we can do all three

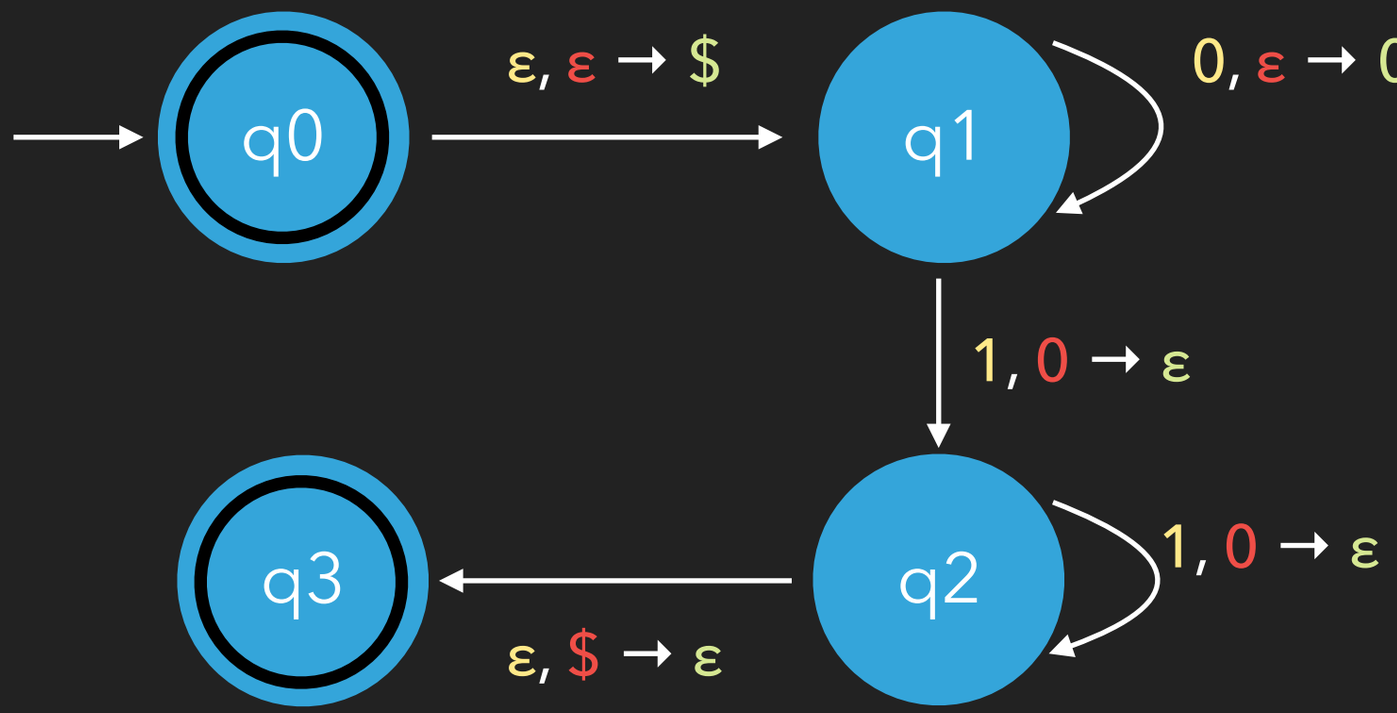
PUSHDOWN AUTOMATA

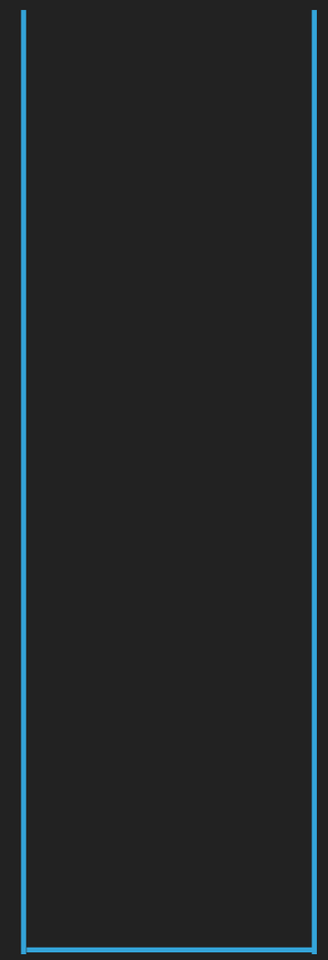
Σ: { 0, 1 }

Γ: { $, 0 }

The stack alphabet consists of things we need to keep track of. The bottom of the stack ($) and each zero we've seen (0).

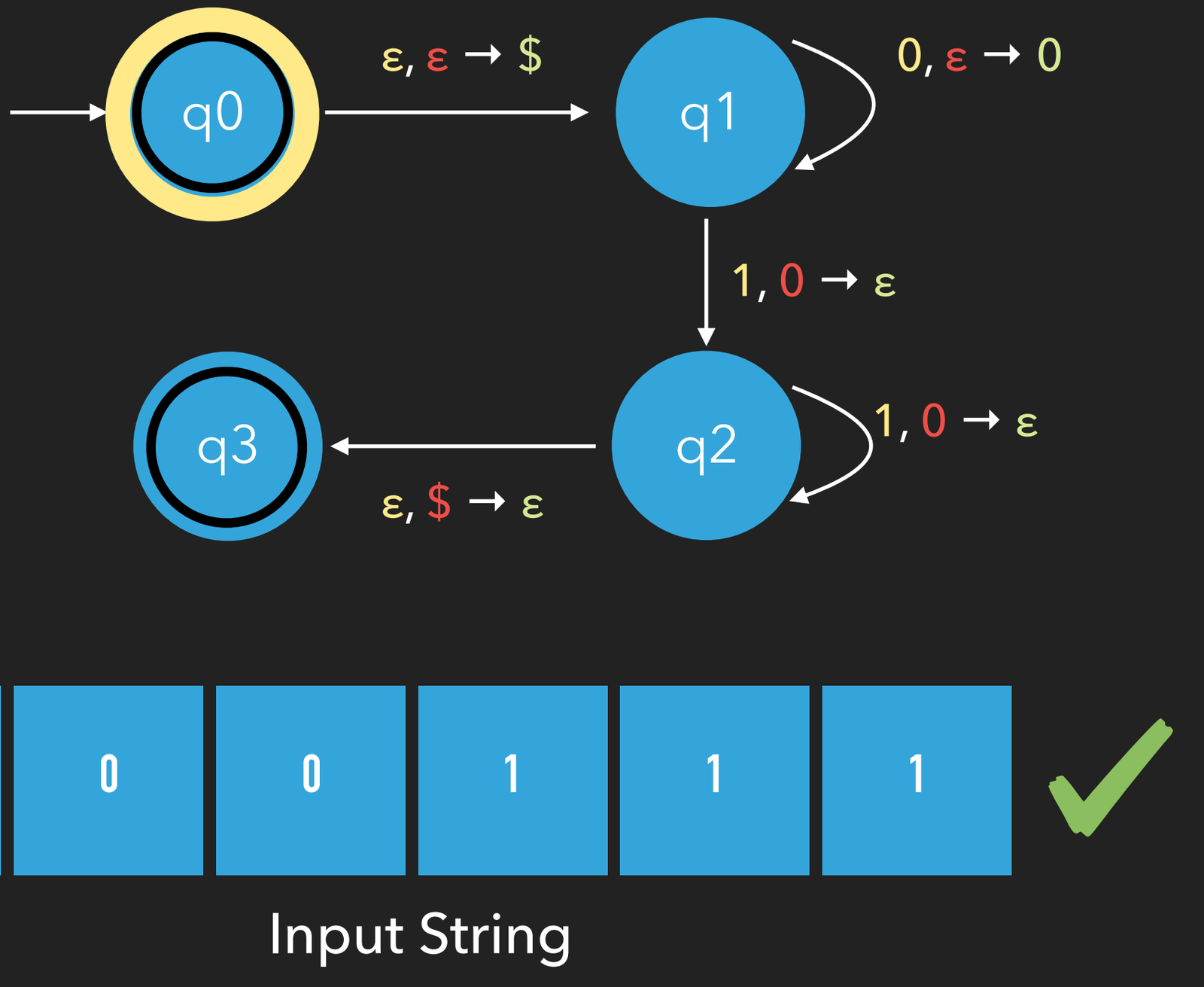
$L = \{ 0^n 1^n \mid n \in \mathbb{N} \}$





PUSHDOWN AUTOMATA

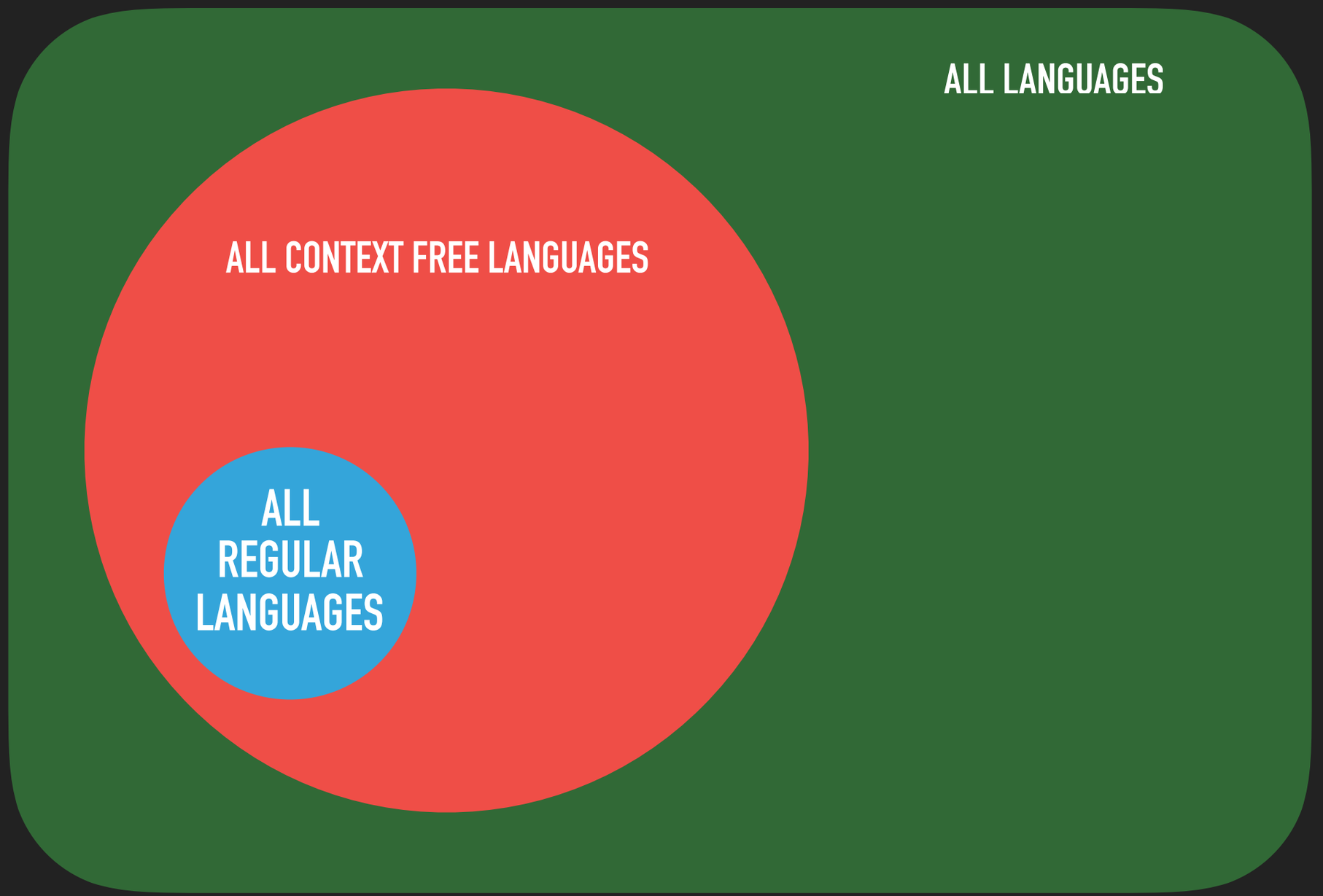
$$L = \{ 0^n 1^n \mid n \in \mathbb{N} \}$$



PUSHDOWN AUTOMATA AND CFLS

Any Context-Free Language can be recognized by a Pushdown Automata.

Any Regular Language can be recognized by a Pushdown Automata.

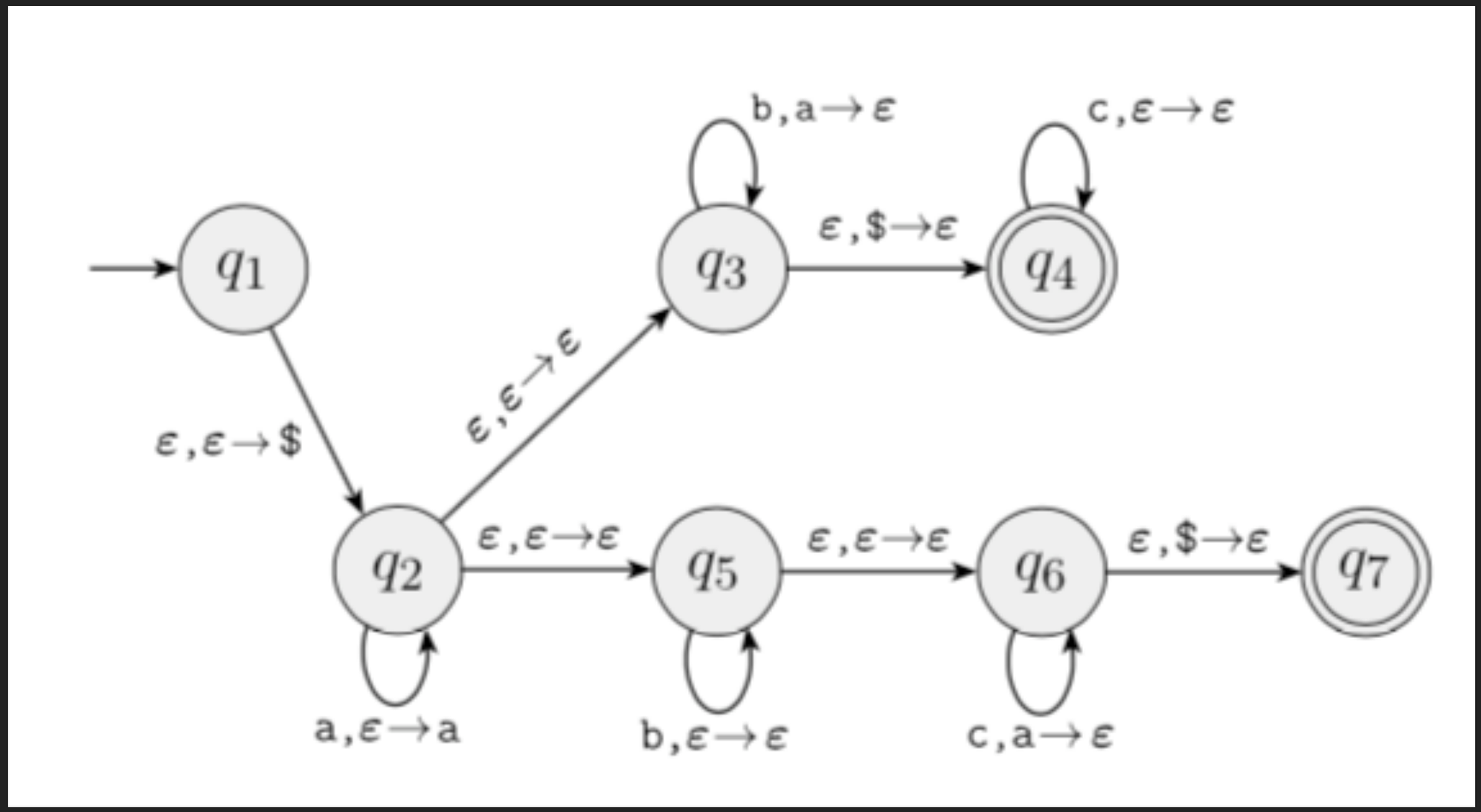


PUSHDOWN AUTOMATA

$$L = \{ a^i b^j c^k \mid i, j, k > 0 \text{ and } i = j \text{ or } i = k \}$$

aaabbbc

aaaabcccc



PUSHDOWN AUTOMATA

$$L = \{ ww^R \mid w \in \{0, 1\}^* \}$$

w “reversed”

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