CSC 240

REGULAR EXPRESSIONS 2

OPERATIONS ON REGULAR LANGUAGES (THE REGULAR OPERATIONS)

Let A and B be regular languages:

Union: $A \cup B = \{ x \mid x \in A \vee x \in B \}$

Concatenation: $A \circ B = \{ xy \mid x \in A \wedge y \in B \}$

Star: $A^* = \{ x_1 x_2 \ldots x_k \mid k \geq 0 \wedge \text{each } x_i \in A \}$

Usually written
AB

Examples:

$\Sigma = \{a \ldots z\}$

$A = \{ \text{happy}, \text{sad} \}$

$B = \{ \text{cat}, \text{dog} \}$

$A \cup B = \{ \text{happy}, \text{sad}, \text{cat}, \text{dog} \}$

$A \circ B = \{ \text{happycat}, \text{sadcat}, \text{happydog}, \text{saddog} \}$

$A^* \{ \varepsilon, \text{happy}, \text{sad}, \text{happyhappy}, \text{sadsad}, \text{happysad}, \text{sadhappy}, \text{sadsadhappy}, \text{sadhappysad}, \ldots \}$

CONCATENATION SYNTAX

$\Sigma = \{a\ldots z\}$

$A = \{ a, b \}$

$AA = \{ aa, ab, ba, bb \}$

$AAA = \{ aaa, aab, aba, baa, bba, bab, abb, bbb \}$

$A^0 = \{\varepsilon\}$

$A^1 = A$

$A^2 = AA$

$A^* = \{ x_1x_2\ldots x_k \mid k \geq 0 \wedge \text{each } x_i \in A\}$

$A^+ = \{ x_1x_2\ldots x_k \mid k \geq 1 \wedge \text{each } x_i \in A\}$

Sometimes called the "Kleene Star"

Sometimes called the "Kleene Plus"

- A language is called a regular language if some finite automaton recognizes it.
- Every nondeterministic finite automaton has an equivalent deterministic finite automaton.
- A language is regular if some nondeterministic finite automaton recognizes it.
- A language is regular if and only if some regular expression describes it.

WAYS TO DETERMINE IF A LANGUAGE IS REGULAR

- If we can create a DFA that recognizes L.
- If we can create an NFA that recognizes L.
- If L can be formed from other regular languages using regular operations.

Regular Expressions are expressions built from regular operations and are used to describe a language.

R is a regular expression if R is:

a for some a in Σ

ε

$\varnothing$

$(R_1 \cup R_2)$ Where R_1 and R_2 are regular expressions

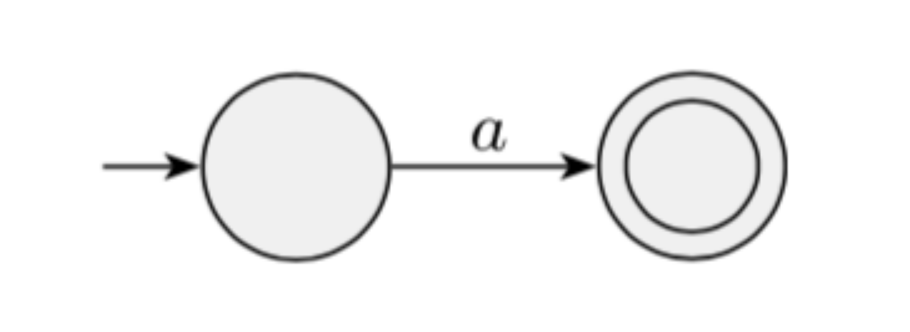
$(R_1 \circ R_2)$ Where R_1 and R_2 are regular expressions

$(R_1{}^*)$ Where R_1 is a regular expression

CONVERTING REGULAR EXPRESSIONS INTO NFAS

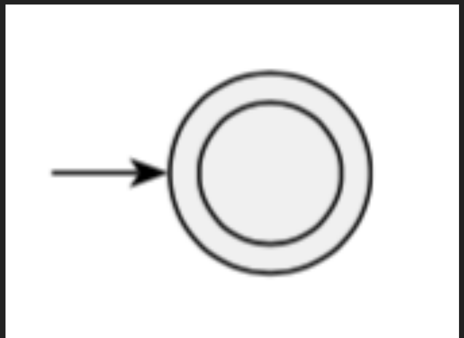
R = a for some a in Σ

L(R) = {a}



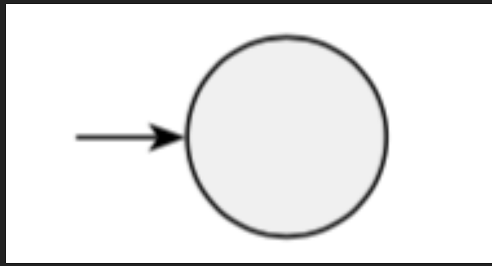
R = ε

L(R) = {ε}



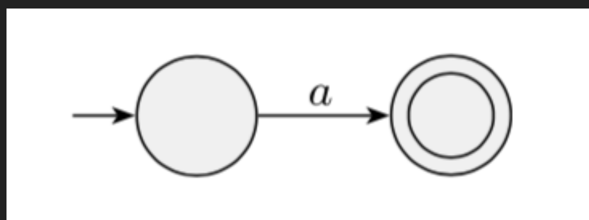
R = ∅

L(R) = ∅

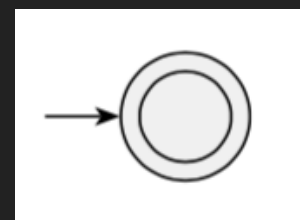


CONVERTING REGULAR EXPRESSIONS INTO NFAS

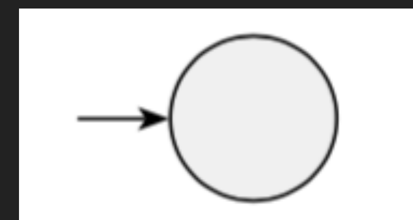
$L(R) = \{a\}$



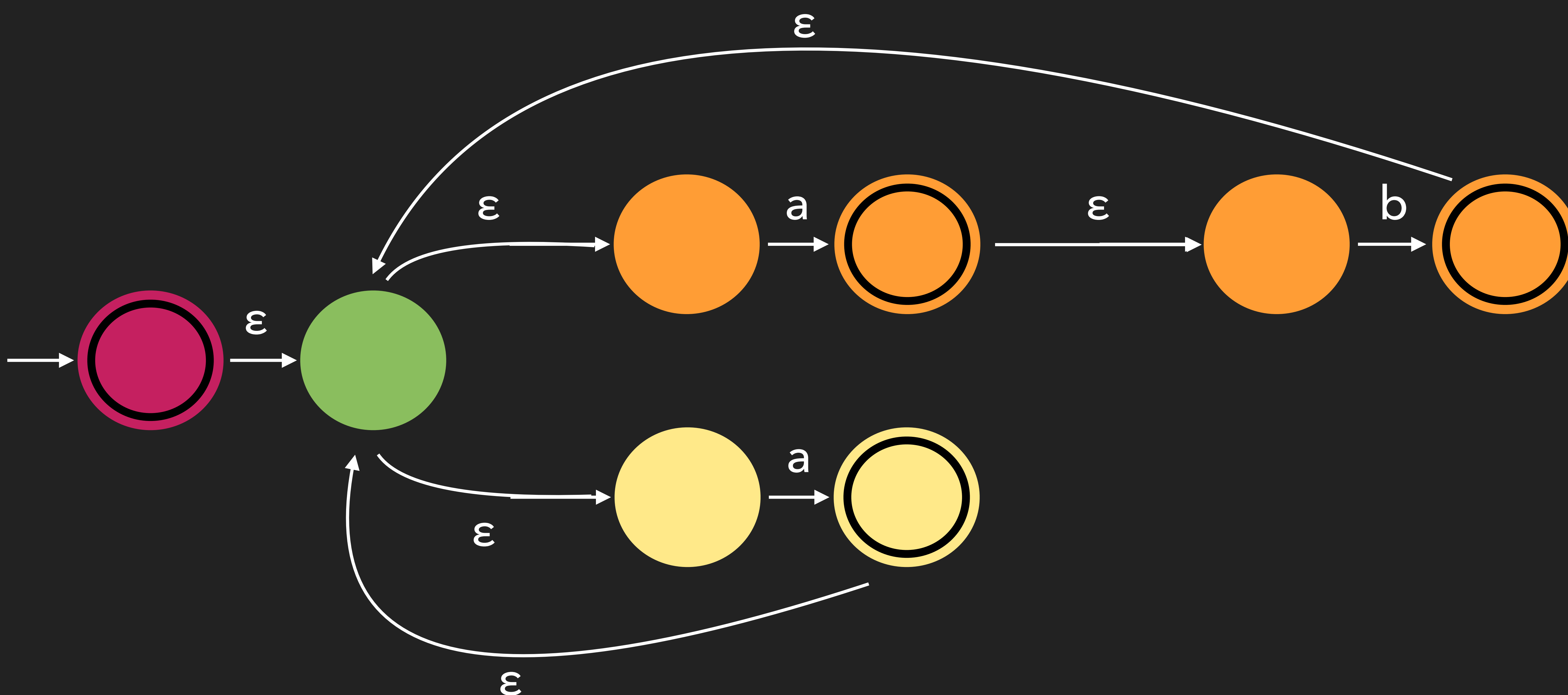
$L(R) = \{\varepsilon\}$



$L(R) = \varnothing$

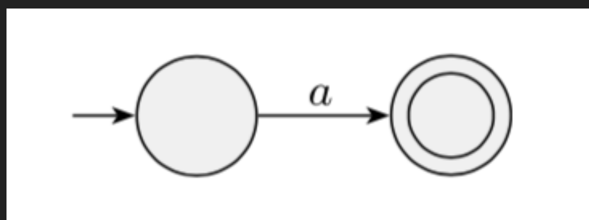


$R = (\,ab \cup a)^*$

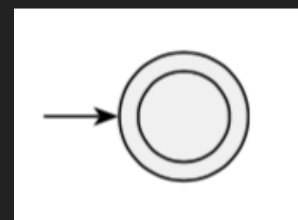


CONVERTING REGULAR EXPRESSIONS INTO NFAS

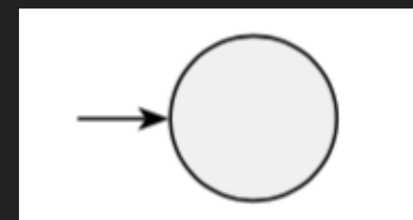
$L(R) = \{a\}$



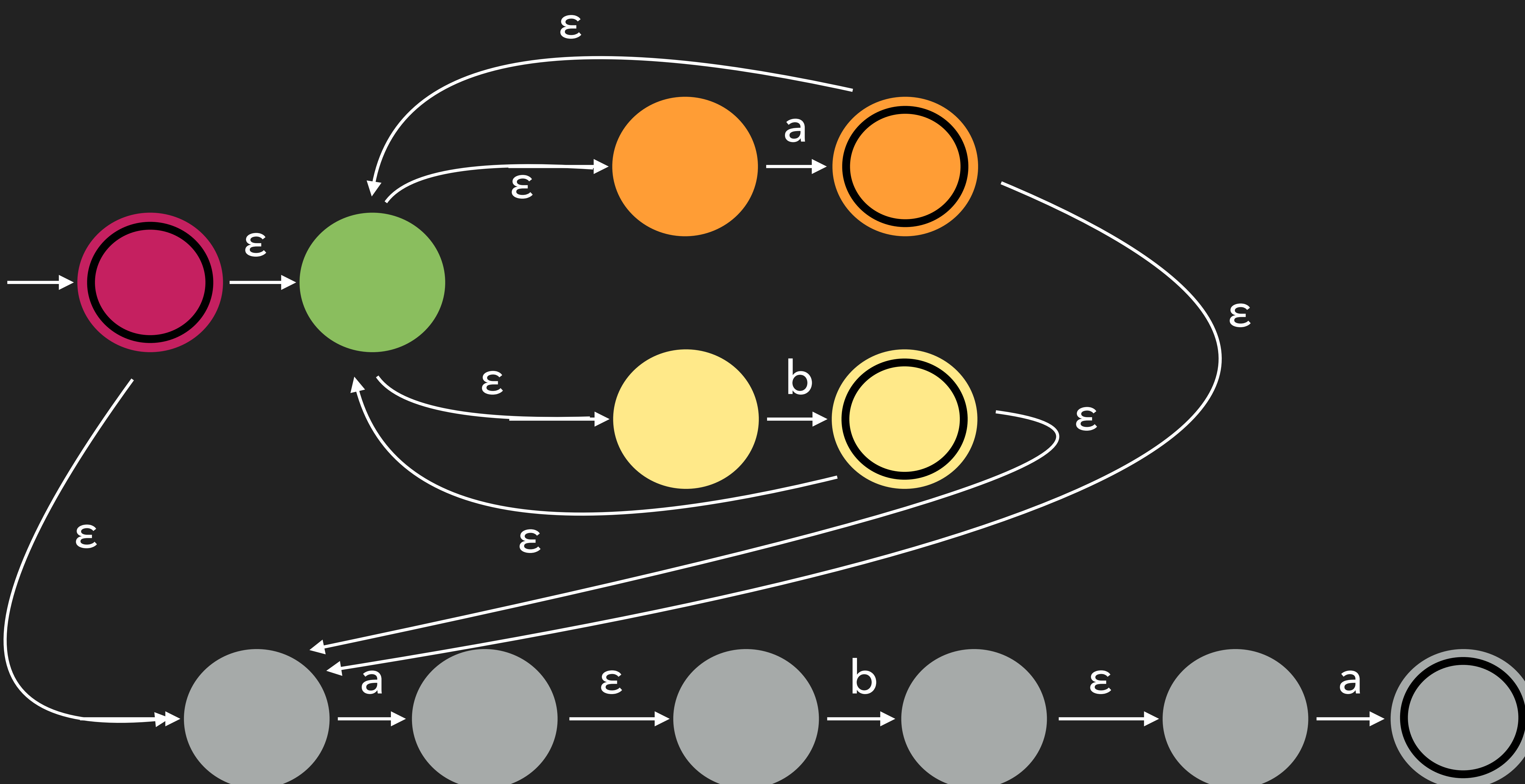
$L(R) = \{\varepsilon\}$



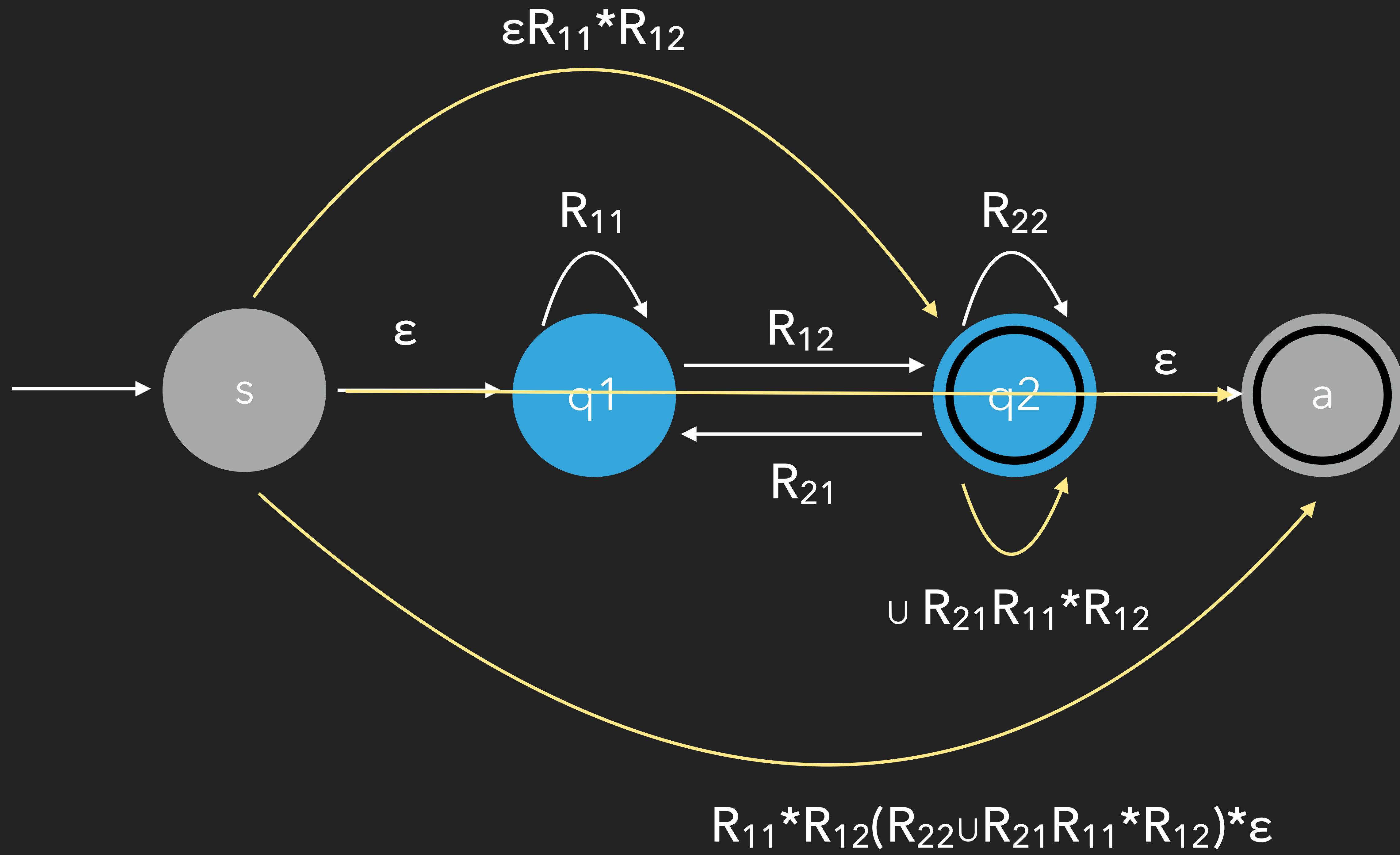
$L(R) = \emptyset$



$R = (a \cup b)^*aba$



CONVERTING FINITE AUTOMATA INTO REGULAR EXPRESSIONS



These statements are equivalent:

X is a regular language

There is a DFA M such that $\mathscr{L}(\mathrm{M}) = \mathrm{X}$

There is an NFA N such that $\mathscr{L}(\mathrm{N}) = \mathrm{X}$

There is a Regular Expression R such that $\mathscr{L}(\mathrm{R}) = \mathrm{X}$