CSC 240

FINITE AUTOMATA

Computers Are Complex Machines

How can we Model Them?

AUTOMATON

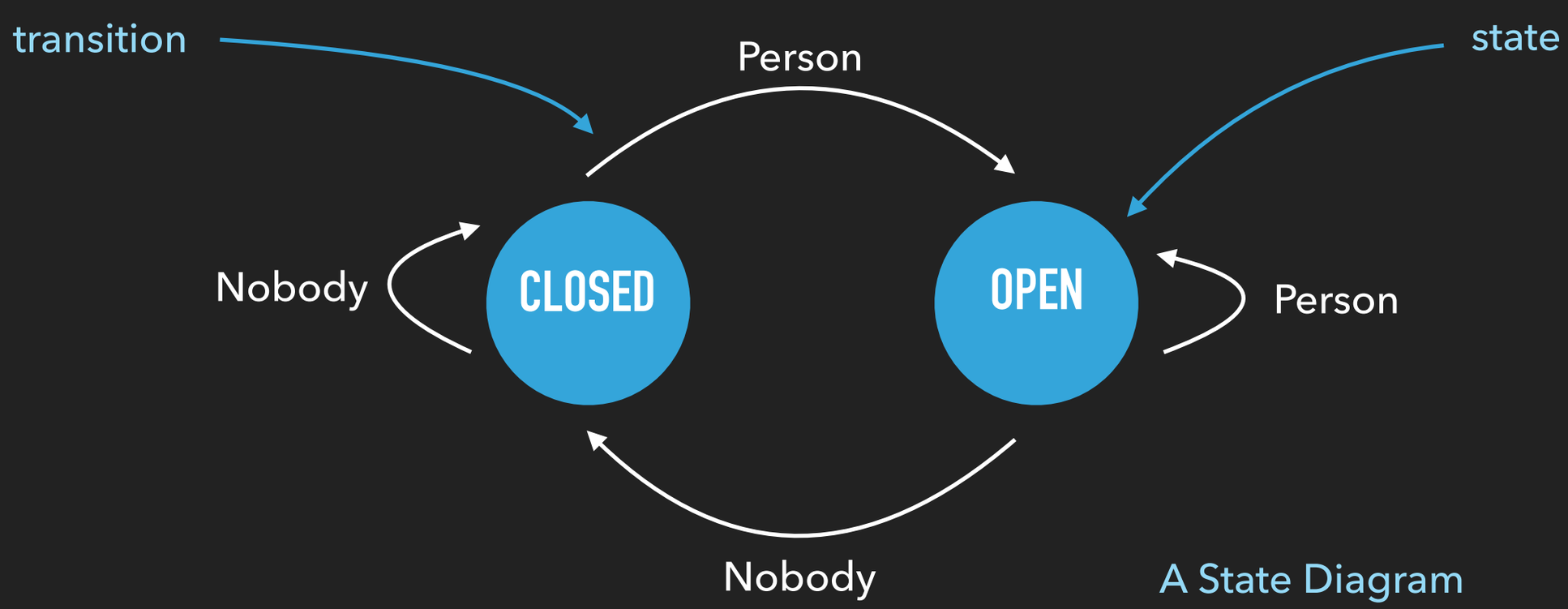
automaton | ôˈtämədən, ôˈtämәˌtän |

noun (plural **automata** | -tə | or **automatons**)

a moving mechanical device made in imitation of a human being.

- a machine that performs a function according to a predetermined set of coded instructions, especially one capable of a range of programmed responses to different circumstances.
- used in similes and comparisons to refer to a person who seems to act in a mechanical or unemotional way: *she went about her preparations like an automaton.*

AUTOMATON

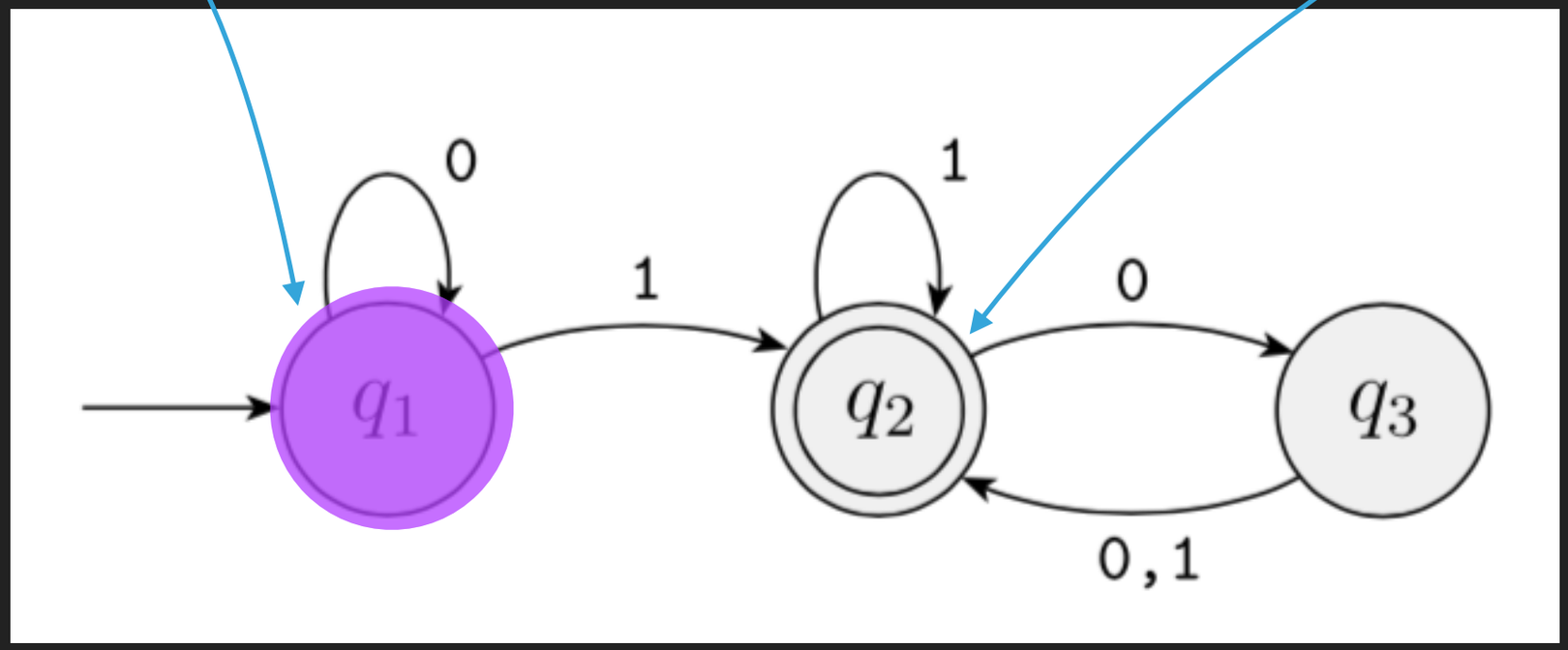


A State Diagram

FINITE AUTOMATA

"Start State"

"Accepting State"



Input: 1101 Accepted

Input: 0010 Not Accepted

This finite automata accepts any string that ends in 1 and any string that ends with an even number of 0's following a 1.

SOME DEFINITIONS

Character: A single symbol.

Alphabet (Σ): A finite, non-empty set of characters.

String Over Alphabet Σ: A finite, sequence of characters drawn from Σ.

Empty String (ε): A string containing no characters.

A Formal Language: A set of strings.

DETERMINISTIC FINITE AUTOMATA (DFA)

- Defined relative to an alphabet.
- Each state has exactly one transition for each symbol in the alphabet.
- Has a unique Start State.
- Has zero or more more accepting states.

DFA - FORMAL DEFINITION

DFA: Defined by a 5-tuple: $(Q, \Sigma, \delta, q_0, F)$

Q: A finite set called states.

Σ: A finite set called the alphabet.

δ: Q x Σ → Q is the transition function.

q_0: is the start state.

F: is the set of accepting states where $F \subseteq Q$.

DFA - FORMAL DEFINITION

Q: A finite set called states.

$Q = \{q_1, q_2, q_3\}$

Σ: A finite set called the alphabet.

$\Sigma = \{0, 1\}$

δ: Q x Σ → Q is the transition function.

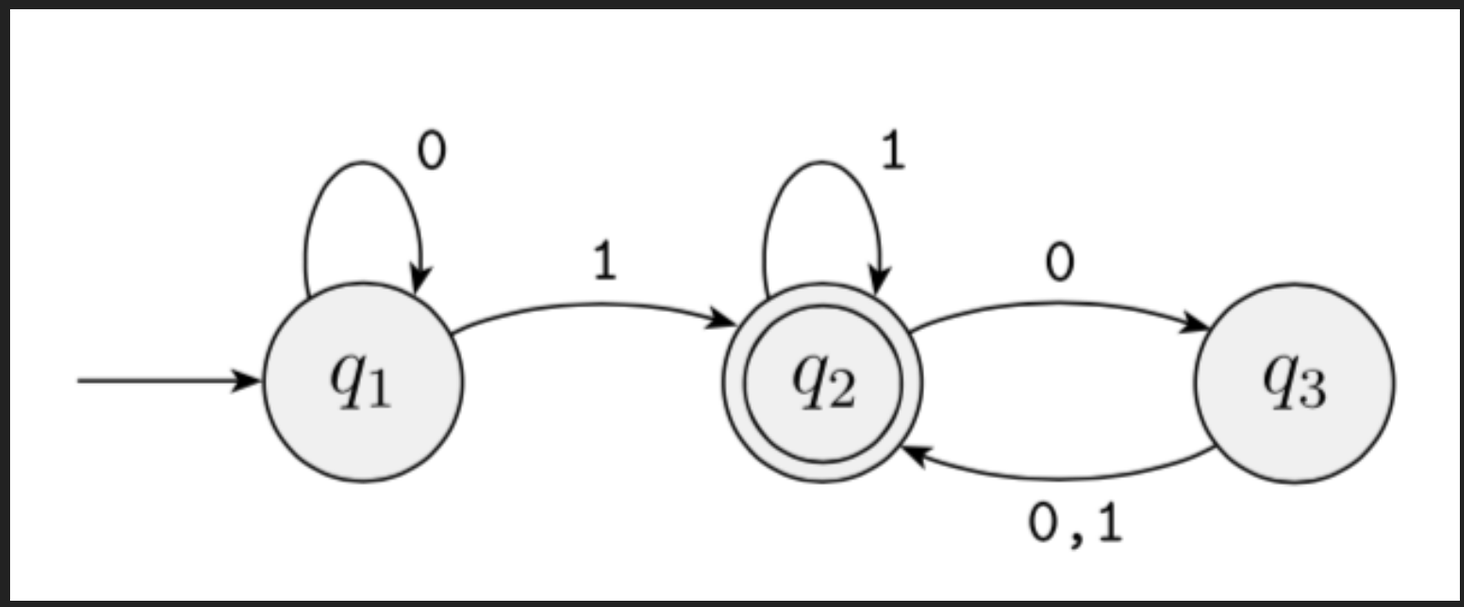
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

q_0: is the start state.

$q_0 = q_1$

F: is the set of accepting states where $F \subseteq Q$.

$F = \{ q_2 \}$



LANGUAGE OF AN AUTOMATA

The Language of an Automata: The set of strings accepted by the automata.

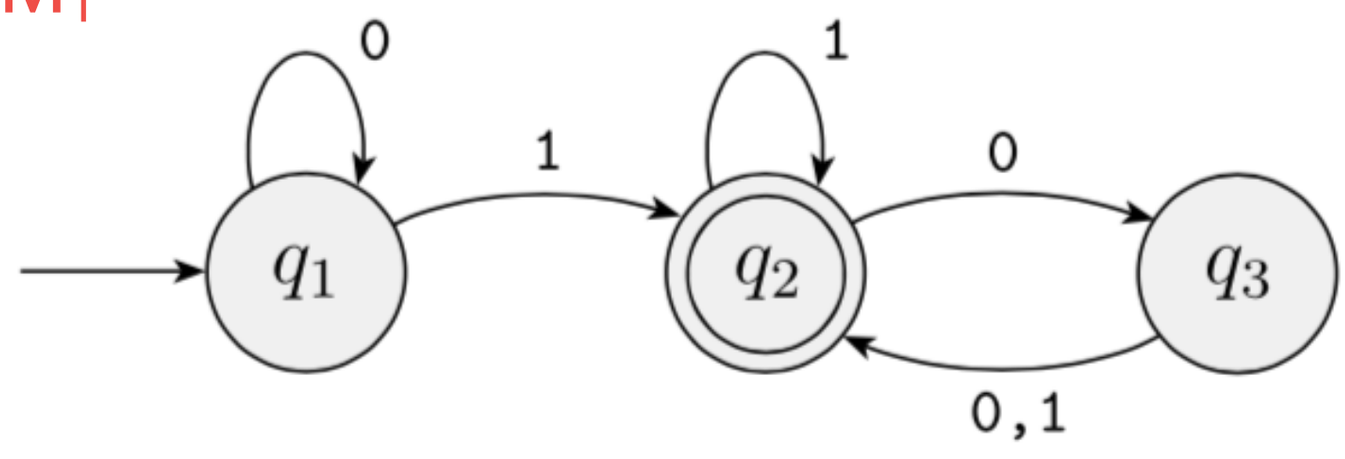
$$\mathscr{L}(M) = A$$

The $\mathscr{L}$anguage of automata M

A is the set of all strings accepted by M.

FINITE AUTOMATA - FORMAL DEFINITION

M_1

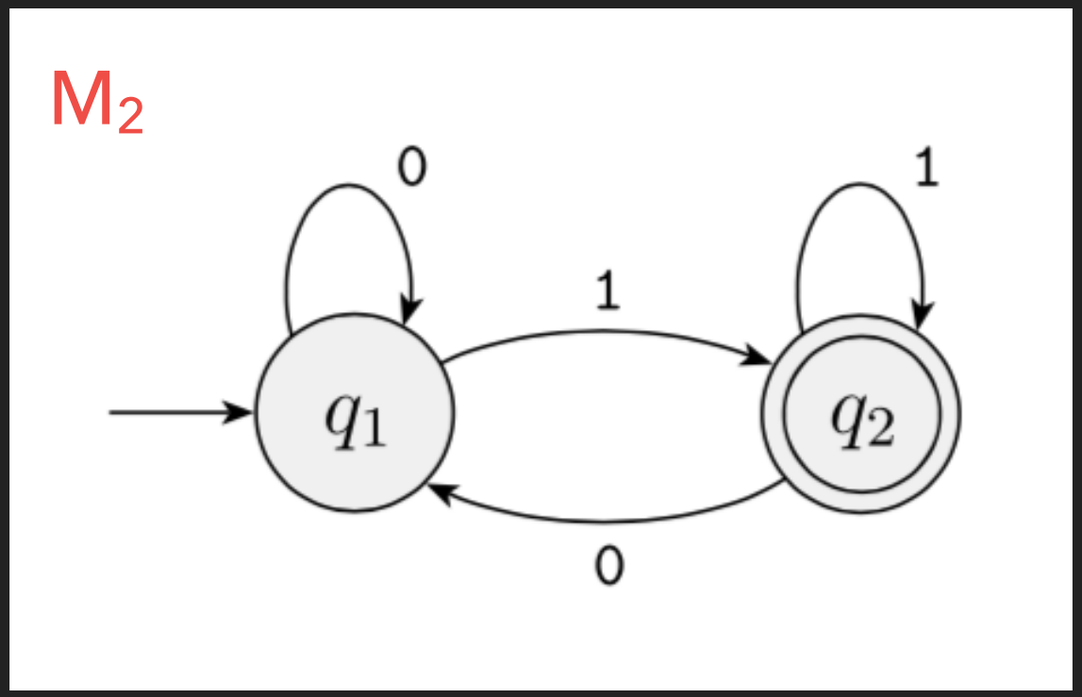


A = { w | w contains at least one 1 and an even number of 0's follow the last 1 }

$\mathscr{L}(M_1) = A$

M_1 *recognizes* A

FINITE AUTOMATA



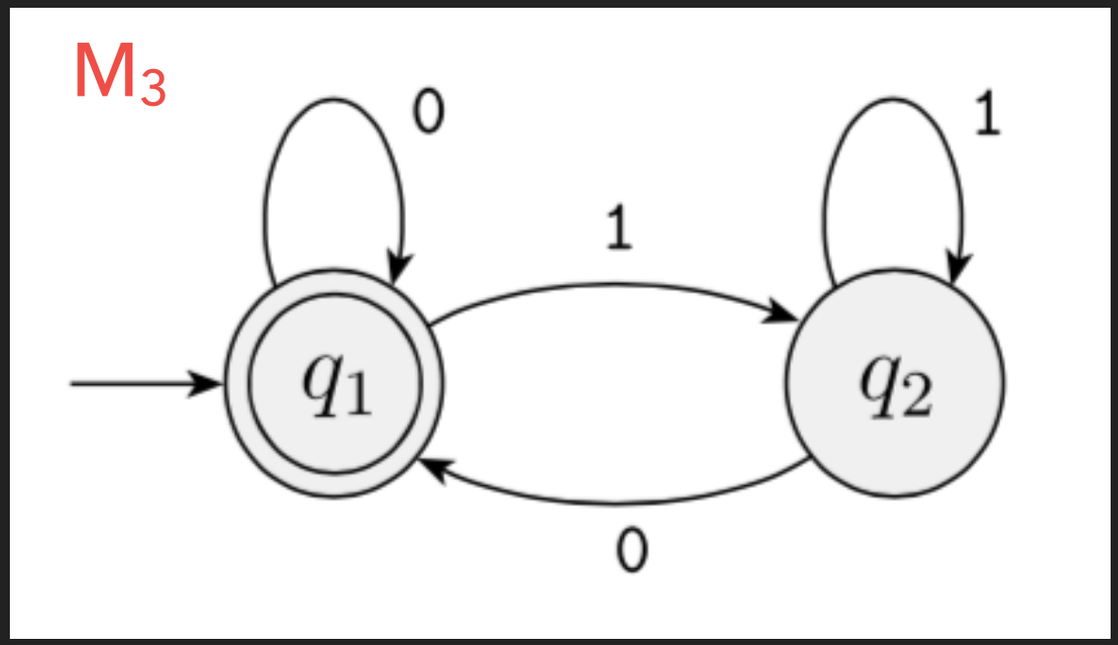
Defined by a 5-tuple: $(Q, \Sigma, \delta, q_0, F)$

$M_2 = \{\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_2\}\}$

δ	0	1
q_1	q_1	q_2
q_2	q_1	q_2

$\mathscr{L}(M_2) = \{ w \mid w \text{ ends in } 1 \}$

FINITE AUTOMATA



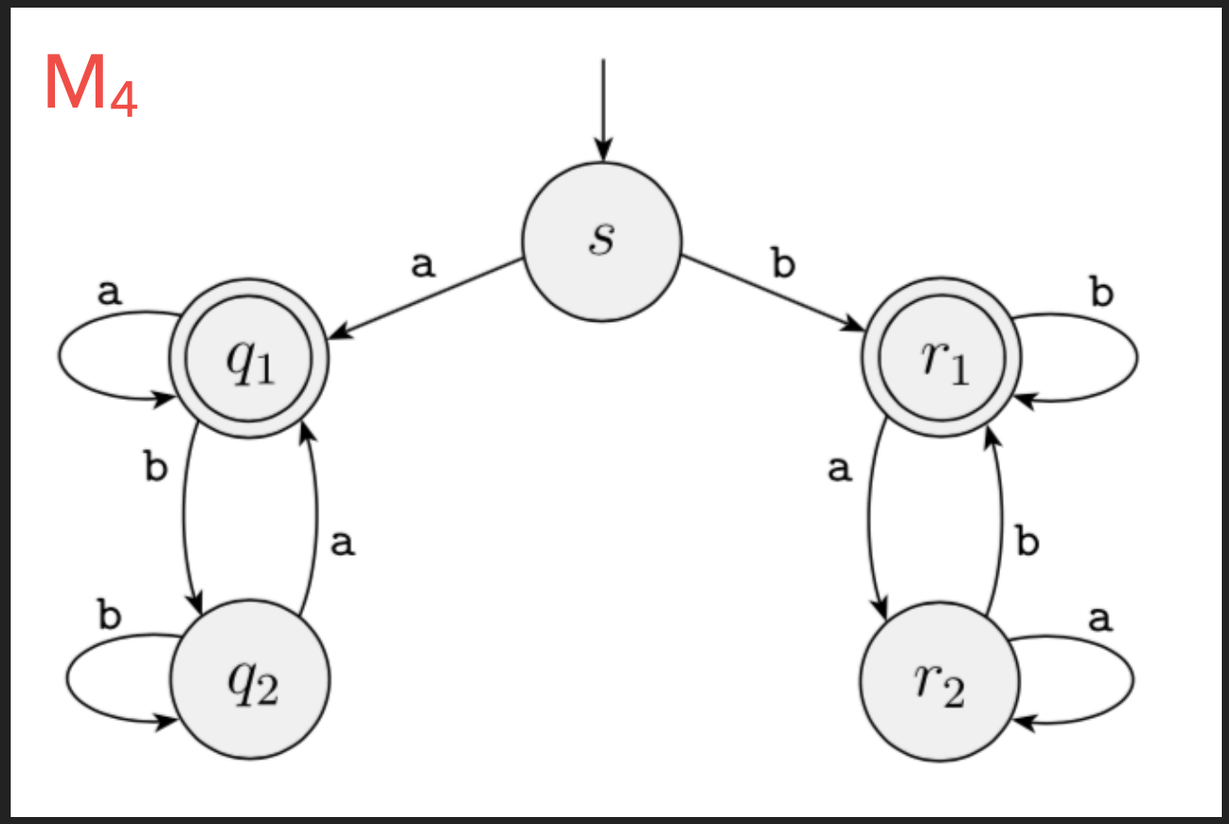
Defined by a 5-tuple: $(Q, \Sigma, \delta, q_0, F)$

$M_3 = \{\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\}\}$

δ	0	1
q_1	q_1	q_2
q_2	q_1	q_2

$\mathcal{L}(M_3) = \{ w \mid w \text{ ends in 0 or } w \text{ is the empty string } \varepsilon\}$

FINITE AUTOMATA



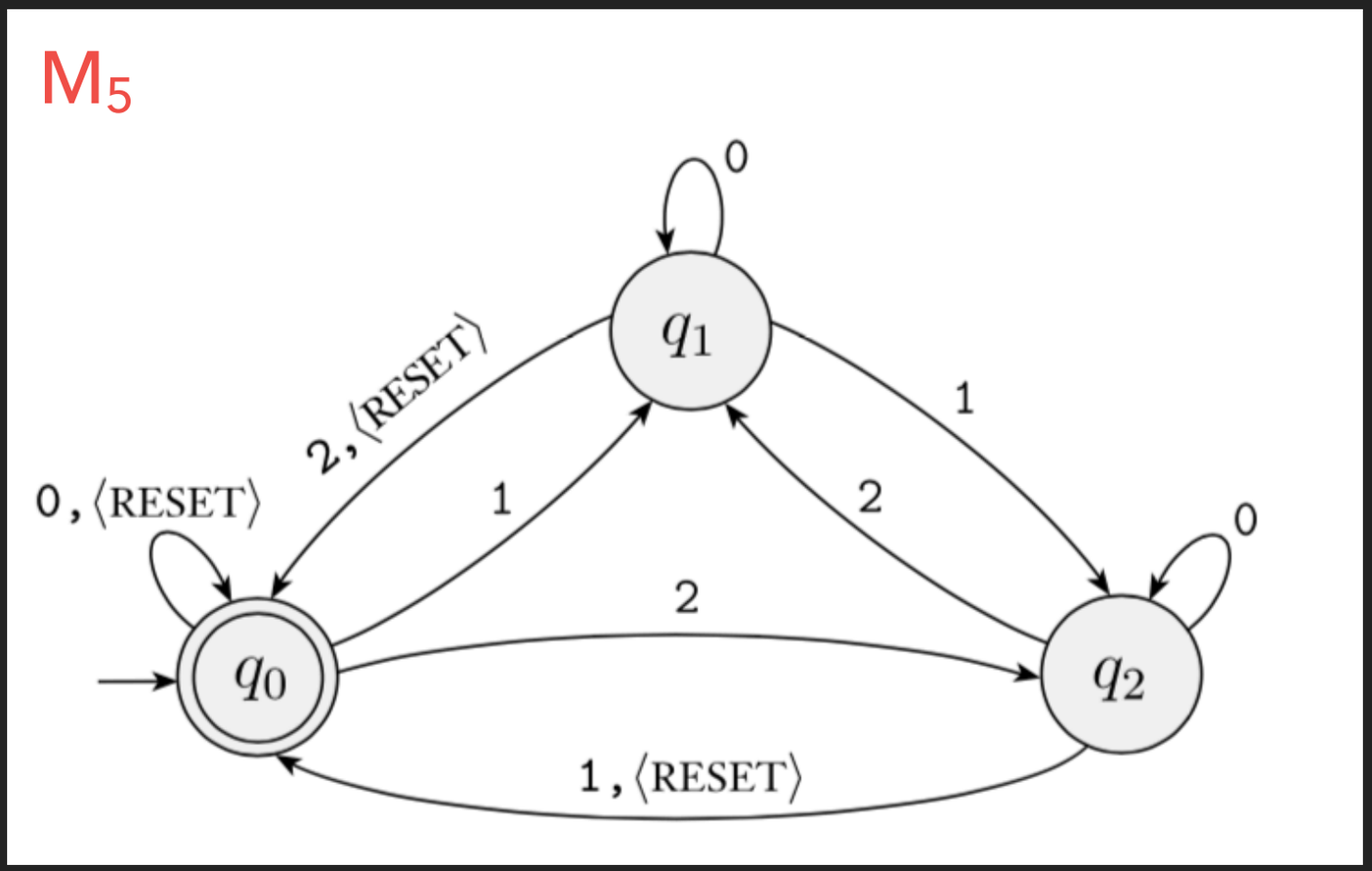
$M_4 = \{\{s, q_1, q_2, r_1, r_2\}, \{a, b\}, \delta, s, \{q_1, r_1\}\}$

$\mathscr{L}(M_4) = \{ w \mid w \text{ start and end with the same symbol}\}$

δ	a	b
s	q_1	r_1
q_1	q_1	q_2
q_2	q_1	q_2
r_1	r_1	r_2
r_2	r_2	r_1

FINITE AUTOMATA

M_5



$\Sigma = \{0, 1, 2, \langle RESET \rangle\}$

Input: 012 Accepted

Input: 1111 Not Accepted

Input: 11112 Accepted

$\mathscr{L}(M_5)$ = { w | the sum of the digits in w is evenly divisible by 3 where <RESET> resets the sum to 0}